Risk publications

Alternative premia

Special report

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Navigating alternative premia

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A growth industry

While attracting considerable attention in academia and within financial institutions, the assets under management (AUM) of factor investing strategies – or at least those explicitly recognised as such – had been rather limited until recently. It is only in the last decade that this has changed. The growth of the industry in recent years has been explosive, with new providers and funds continually appearing. Between 2013 and 2018, the majority of the providers (both within banks and asset management firms) saw significant AUM growth. From the niche it once occupied, the factor investing sector has boomed to manage more than an estimated USD1 trillion today.

The reasons for this rapid rise are manifold, including the launch of the first alternative premia (AP) solutions in 2013. AP give investors the opportunity to take exposure to well-known risk and style factors in a way only hedge funds could have offered previously. In fact, they can be considered alternatives to traditional hedge funds. The approach is also appealing because the foundations of factor investing are solidly anchored in academic theory going back to the 1970s. Additionally, recent advances in technology have resulted in more capable handling of large datasets and faster computing of portfolio optimisations. Finally, the under-performance, on average, of actively managed funds and their unsatisfying levels of correlation with traditional asset classes (and equities in particular) has also driven inflows into AP strategies.

The way forward

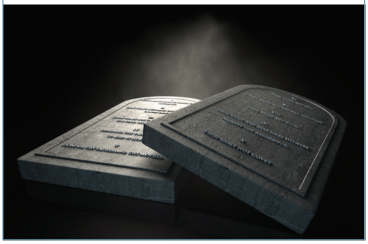
Despite the solid growth, the industry is facing some technical challenges and investor concerns.

There is often a confused perception of the concepts on which the industry is based. A non-uniform terminology across different providers can create misunderstandings with investors. AP themselves are known by half a dozen different names, often resulting in unclear communication.

The proliferation of players and products can make selecting the appropriate provider and solution a challenge. Providers may offer products that are nominally similar to one another other, deploying analogous strategies to the same asset class. For an investor, it is often difficult to distinguish, for example, between AP and an equity portfolio managed with a value strategy. The implementation of strategies by different providers often differs significantly, impacting performance as a consequence and generating what is known as performance dispersion – a common concern among investors. In the specific case of equity, the performance may be affected by different instruments used for hedging or the use of different predictive indicators. This proves that a factor approach is only a framework for investing and the ability to implement a strategy and optimise its performance is key to a successful manager and AP provider.

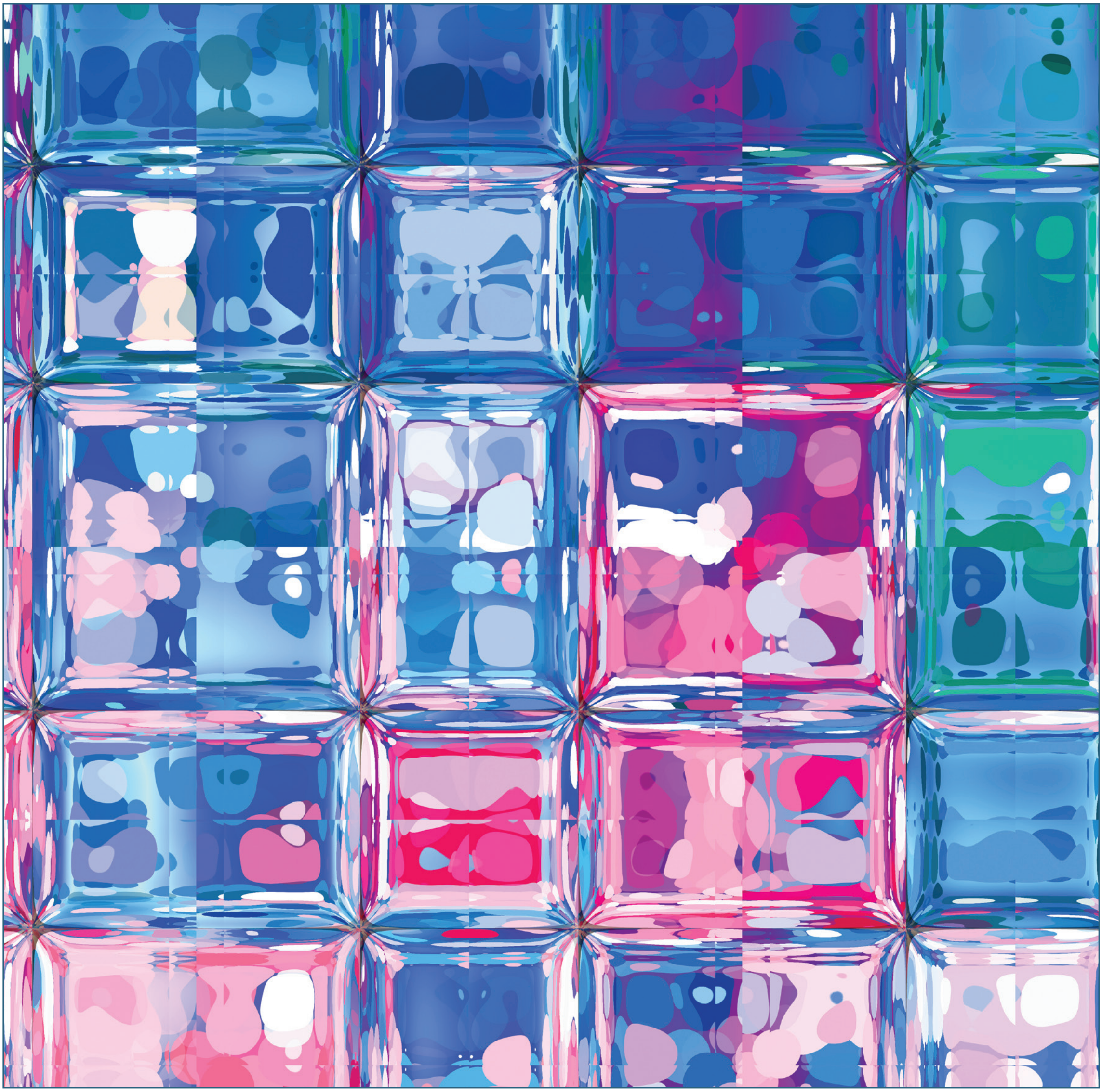
More to the core of the business, the proliferation of factors, which has run at the rate of tens of new factors per year over the past decade, has not only made the choice among investment opportunities more complex, but has also shown little relation to positive performance. AP managers are often asked about the risks of overcrowding – the risk of performance deteriorating if excessive capital is invested in a single strategy – and recorrelation, with success in multi-factor AP investing depending on the ability to combine uncorrelated strategies.

The research papers that follow aim to address these fundamental issues and help investors to make clear and informed choices when it comes to AP investing.



Premia 2.0
An introduction to advanced factor investing

La Française Investment Solutions explains how its highly diversified multi-asset portfolio of risk and style premia can benefit clients beyond traditionally implemented alternative factor approaches



The death of diversification

The 2007–08 global financial crisis confirmed two things: on average, most funds tend to be exposed – at best, and in benign conditions – to just a few risk factors; and, at worst – and especially in times of risk aversion – to just one, the equity market factor. Allocation funds, whether classified as 'balanced', 'flexible' or 'alternative', have historically posted performances that strongly correlate with equity markets while not outperforming equities on a risk-adjusted basis (figure 1). In other words, the performance of investment funds is, on average, highly beta-driven.

This phenomenon is well documented in research by Ang, Goetzmann and Schaefer,[1] who studied active management at the Norwegian Government Pension Fund Global (GPFG), one of the largest sovereign wealth funds in the world. Notwithstanding GPFG's sophisticated investment infrastructure and highly qualified investment staff, it turned out that exposure to the equity market factor – in effect, equity risk – accounted for more than 70% of the portfolio's returns and was the main driver behind a 23.3% loss in 2008.

This explains the emergence, which began in 2008, of risk parity solutions that attempt to 'force' effective diversification of the equity factor through an equally risk-weighted allocation to equities and government bonds. However, a risk parity allocation – which corresponds to a capital allocation of 15% to equities and 85% to bonds – has lost appeal in today's context of globally low yields.

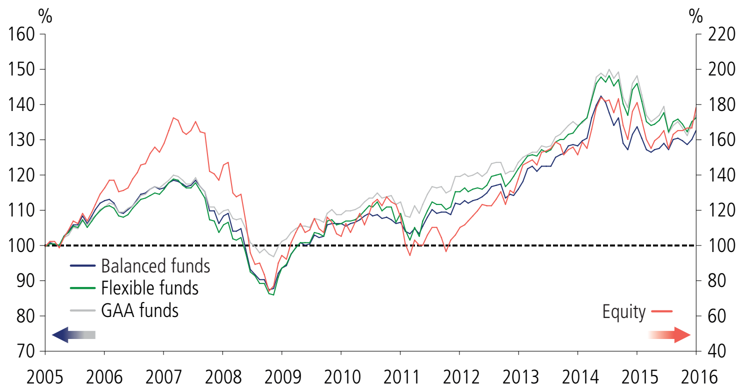
The secret to success

Certain funds and managers have succeeded in beating their benchmarks. When the components of their returns are assessed, research reveals that these funds are exposed to additional factors besides the equity market factor.

These 'alternative factors' were documented by Nobel laureate Eugene Fama and Kenneth French, both professors at the University of Chicago Booth School of Business. Their Fama French three factor model separates equity returns into three distinct risk factors:

- Equity market risk;
- The value factor – buying the cheapest stocks and selling the most expensive, based on their valuation multiples; plus
- The size factor – buying the smallest capitalisation stocks and selling those with the highest capitalisation.

Figure 1 All beta all the time



Equity – MSCI World TR USD
Flexible funds – Bloomberg's Active Indices for Funds (BAIF) – Ucits/open-ended/flexible funds
Global asset allocation (GAA) funds
Balanced funds – BAIF – Ucits/open-ended/balanced funds

Source: Bloomberg, La Française Investment Solutions

Smart beta but still beta

Smart beta solutions seek to exploit these factors to drive returns in a long-only way. A smart beta fund will, for example, buy the most undervalued small caps to outperform its investment universe. However, this approach remains long, and the underlying market and performance is still irredeemably linked to that of equities.

An alternative premia approach, on the other hand, involves the simultaneous purchase of the most attractive stocks and the sale of the less attractive (figure 2).

By design, an alternative premia approach has an additional performance driver, capturing not only the 'long' leg of typical smart beta linked to the outperformance of the most attractive stocks, but also a 'short' leg linked to the underperformance of less attractive stocks. An alternative premia approach therefore effectively neutralises the beta exposure of the portfolio. Long and short exposure to markets cancel out for a truly market-neutral approach.

Expanding the use of the alternative premia framework

The traditional premia approach is focused on the standard factors – such as value, carry and momentum – within traditional asset classes and equities in particular. The definition of alternative premia can, however, be expanded to other factors and asset classes, including implied assets such as volatility and dividends and pure arbitrage strategies such as repo, negative basis and convertible arbitrage strategies.

The broad alternative premia universe can be broken down into two types, each with a specific underlying structural rationale. Risk premia include strategies that remunerate investors for exposure to an additional systemic – economic or financial – risk factor that cannot be diversified away. Style premia remunerate investors for capacity, cash or regulatory, to implement strategies that profit from structural market biases, whether behavioural or linked to investment constraints and structural flows.

The foundations of alternative premia strategies are likely to persist. Rational investors will always require returns to take on additional systemic risk. In the same manner, behavioural biases are so strongly ingrained among most market participants that it will always prove difficult for rational investors to arbitrate between them completely. Finally, the raft of regulation that applies to most actors in financial markets (the Basel Accords for banks and Solvency II directives

[1] *A Ang, W Goetzmann and S Schaefer, December 2009,* Evaluation of active management of the Norwegian government pension fund – global, *https://bit.ly/2OVXqcZ*

for insurance companies) is moving the industry towards more rules and stricter enforcement rather than the opposite. This should also generate more opportunities.

Alternative premia – Promising portfolio building blocks

Alternative premia exist in all asset classes; the rationale behind each is different and, as previously highlighted, is likely to persist over time. Specific alternative premia are driven by different underlying factors, so they will materialise at different points in time. A diversified portfolio of multiple alternative premia therefore has the potential to deliver strong, risk-adjusted, market-neutral performance over time.

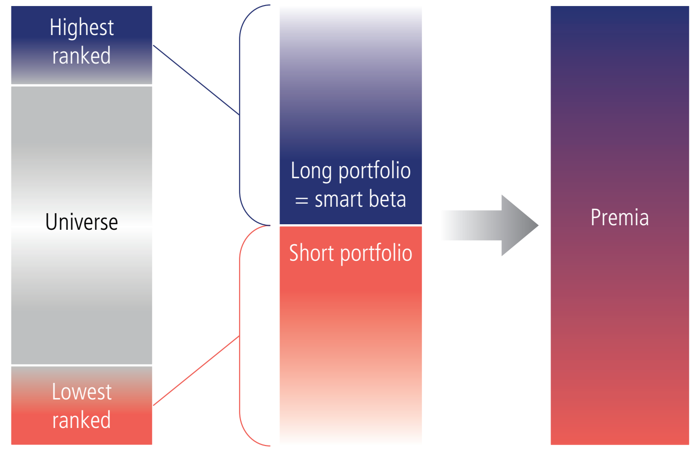
However, the 'alternative premia' label should be thought of as an analysis framework rather than a standalone investment strategy. In summary, while the alternative premia investing concept is very attractive on paper, in practice the pitfalls are numerous, and actual performance may differ greatly from historical simulations. The robustness of any alternative premia solution depends on the choices made by the team in charge of implementation. Whether a provider (portfolio manager) or a user (client or consultant), it is crucial to adopt a comprehensive, forward-looking approach.

Beyond the academic

La Française Investment Solutions' (LFIS's) alternative premia strategy is differentiated by the breadth of investable alternative premia it combines. This approach goes beyond 'traditional' alternative premia to combine approximately 30 strategies across three families:

- **Academic premia** – the most common premia, including value, momentum, carry, low-risk/low-quality and liquidity strategies across the range of asset classes including equities, bonds, currencies and commodities.
- **Implied premia** – parameters include volatility, correlation, dispersion and dividends, and are created by asymmetries in risk and return and specific flows linked to certain investor patterns, hedging by banks, insurance companies, and so on, and regulatory constraints.
- **Liquidity/carry premia** – LFIS's cash capacity and setup allow the holding of certain liquid assets that other market actors can no longer carry, often for regulatory reasons.

Figure 2 From smart beta to premia – A market-neutral, long-short approach



Source: La Française Investment Solutions

Ensuring effective diversification

Diversity across alternative premia strategies is just the beginning. To ensure optimal diversification, it is essential to manage correlation, particularly on the downside. LFIS's allocation and risk management approach considers correlation at every level. The allocation is defined using an equal risk contribution framework to take into account correlation without overemphasising it relative to volatility. Risk management is comprehensive and includes stress tests and drawdown controls based on real market events and bespoke scenarios, concentration limits at all levels (per asset class/sector/underlying/counterparty/issuer) and formal Greek limits, including delta, gamma and vega.

LFIS's comprehensive approach is possible thanks to the background of its portfolio management team, which combines extensive experience in investment banking and quantitative asset management. This, alongside an institutional setup that includes an extensive set of International Swaps and Derivatives Association (Isda) agreements and proprietary tools for pricing over-the-counter (OTC) instruments, positions LFIS to understand opportunities and dislocations in markets and flows, and identify the resulting opportunities. The same setup allows LFIS to negotiate, price, implement, stress-test and risk-manage strategies that capture resulting alternative premia in a pure, market-neutral manner.

LFIS premia 2.0

The result is a highly diversified multi-asset portfolio of risk and style premia that goes well beyond the alternative beta approaches traditionally implemented. LFIS's approach combines diverse alternative premia; some require markets to move, others rely on stability. Strategies can benefit across markets – volatile, trending, range-bound – and materialise at different points in time. This approach also looks to be carry-positive. Performance does not depend on directional market moves, and there is no structural, directional exposure to underlying asset classes.

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Risk-return dominance of US ‘big five’ a myth, data shows

US stock markets less exposed to a ‘Gafam factor’ than assumed, say Luc Dumontier and Guillaume Garchery of La Française Investment Solutions

Until early 2018, the Gafam stocks – Google, Apple, Facebook, Amazon and Microsoft – were every investor's darlings: lauded for their outstanding stock market performance and credited with driving much of the recent rally in US equity indexes.

Since early February, the mood has soured. The Gafams are now accused of fuelling recent episodes of spiking volatility and plunging markets. But does the data support the consensus? Are US equity markets highly exposed to a 'Gafam factor'? The answer is no. Here's why.

The first reason is that Gafams alone aren't behind the S&P's rise.

In a recent publication provocatively titled *Do Stocks Outperform Treasury Bills?* Hendrick Bessembinder, professor of finance at Arizona State University, showed that over an extended period from 1926 to 2016, 42% (or less than half) of US equities[1] outperformed the risk-free rate[2] and only 4% of these were responsible for all the wealth created.

To put it another way, all the other stocks combined (96% of the universe) only managed to deliver the risk-free rate[2]. Over the long term, and contrary to common perception, only a few stocks offer the upside potential we typically associate with equities.

Over a shorter period, this study seems in line with the popular view that Gafams, and more generally the stocks comprising the tech-heavy Nasdaq Composite index, have been the main contributors to the rally in US equity markets since the end of 2014.

The truth is more nuanced, though.

Gafams[3] and the Nasdaq have certainly delivered exceptional performance – 106.9%[4] and 61.5%[4], respectively, on average between December 2014 and March 2018 (figures 1 and 2) – but they are not alone. Over the same period, 71% of the stocks in the Standard & Poor's 500 index, which itself is up 37.2%[4], and all of the Global Industry Classification Standard sectors, except energy, have delivered performance above the risk-free rate[2].

In terms of contribution, Gafams are the top five drivers of the increase in US equities, but it takes 174 from 601[5] stocks (29%) to explain the overall return of the S&P. That compares with 4% in Bessembinder's study. Even without the Gafam stocks the S&P would still have delivered 30.4%[4]. Without the Nasdaq stocks, the index would have delivered 28.0%[4] (figure 2).

The reason the Gafams' contribution seems low compared with their performance is because their weight in the S&P is limited – 11% on average and 14% at the end of the period.

Gafams boosted the increase in US equities, but they alone did not create it.

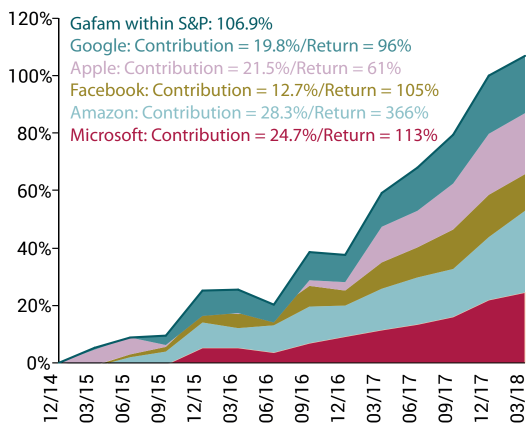
Another common belief is that Gafams alone account for a significant portion of equity market volatility and are responsible for the market dips seen so far this year.

In reality, while the volatility[6] of the Gafam composite is 6% higher than that of the S&P over the past three years (figure 3), most of the difference is due to the diversification effect, which increases mechanically with the number of stocks in the universe. The difference in volatility[6] between the S&P with and without Gafams is only 0.1% on average over the period (the blue area in figure 3). Having said that, the difference reached its highest level of 0.9% at the end of the period.

Gafams did not especially under-perform the S&P during the sharp drops in February. In particular, when the S&P saw its worst loss of the year, –4.1% on February 5, the Gafams lost –3.7%.

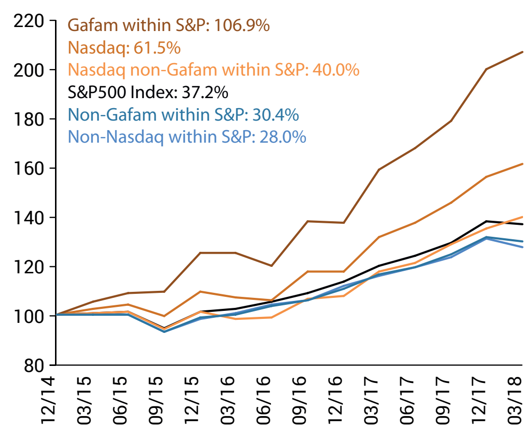
In March, however, the five days on which the S&P lost more than –1% saw the Gafams deliver greater losses. Their particular vulnerability during the second half of March can be attributed to political concerns around their increasing power – US president Donald Trump suggested reviewing the tax treatment of Amazon – and the potential misuse of confidential user information by Facebook.

1. Contributions to cumulative Gafam composite returns



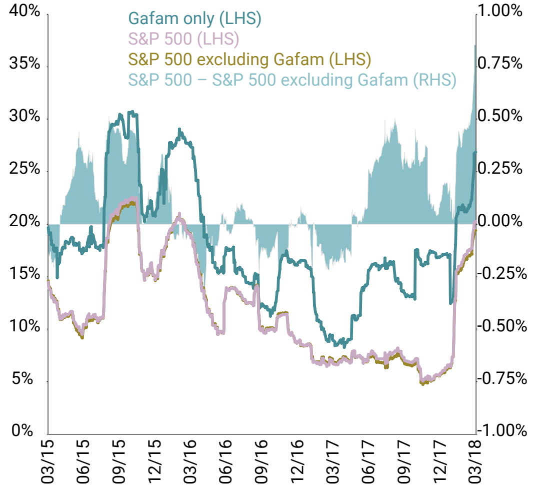
Source: La Française Investment Solutions

2. Cumulated returns of US equity composites



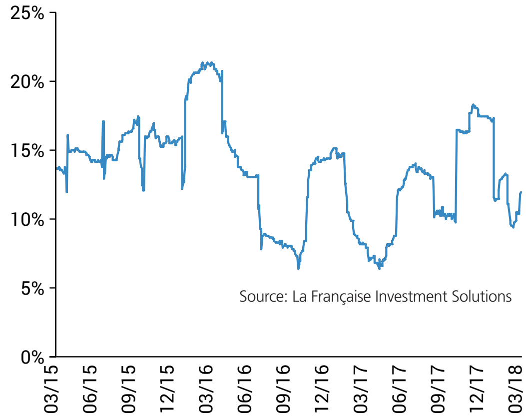
Source: La Française Investment Solutions

3. 60-day rolling volatility of US equity composites

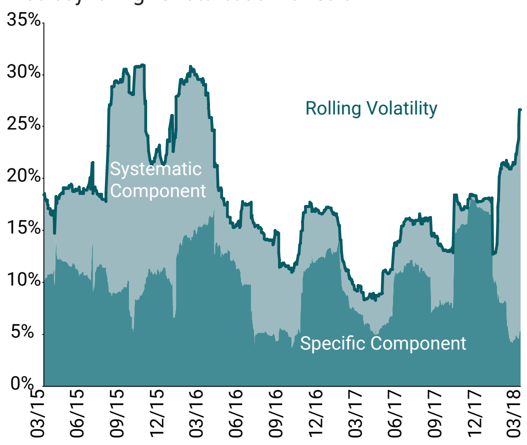


Source: La Française Investment Solutions

5. 60-day rolling specific volatility for Gafam



4. 60-day rolling risk attribution for Gafam



Source: La Française Investment Solutions

Meanwhile, specific risk for Gafam stocks is no higher than for other sectors.

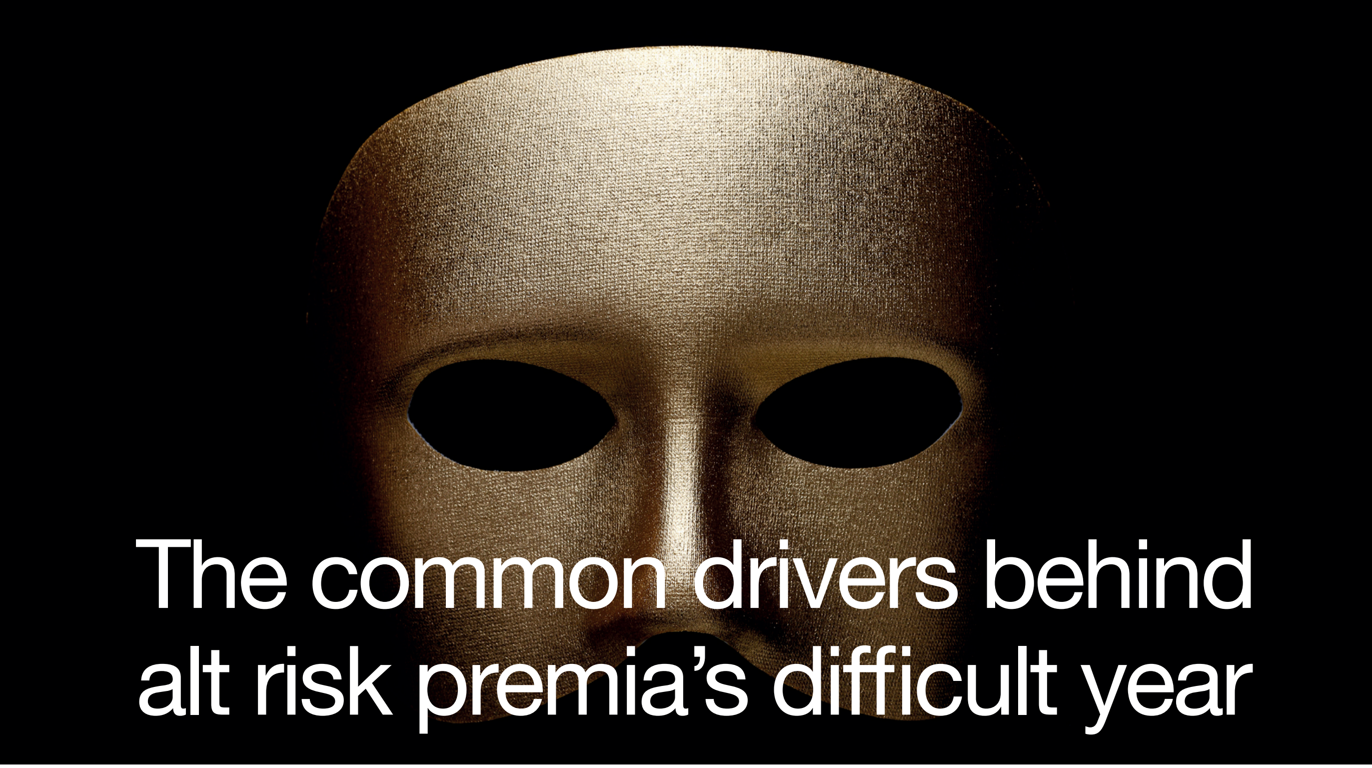
If Gafams represented a significant independent factor, their level of specific risk – risk that does not arise from their exposure to the market – should be a significant part of their total risk. Actually[7], specific and systematic risks explain, on average, 59% and 41% of the total variance of Gafam stocks over the past three years (figure 4). Specific risk is the bigger component, but that is equally true for other sectors. Over the same period, the specific risk component for the five biggest stocks by capitalisation in each of the energy, financials, health care, real estate and consumer staples sectors represented 69%, 49%, 60%, 65% and 64%, respectively, of total risk.

It is also interesting to note that the recent increase in risk for the Gafams is not owed to a spike in specific risk. Specific risk has actually fallen both as a proportion of total risk (figure 4) and in absolute terms (figure 5). The increase in the level of systematic risk for Gafams is primarily due to the increase in risk for the S&P and, to a lesser extent, to their higher beta (1.2 at the end of the period, versus 1 on average).

Gafams' contribution to the risk and return profile of US stock markets is significantly overstated. Of course, that does not mean they are not risky. Their stocks are trading at high valuation multiples. The current price-to-earnings ratio for Amazon, for example, is 130 for 2018. And it is worth noting that such levels will only be justified if profitability increases markedly. That might well depend on the actions of policymakers currently looking at Gafams with the stated objective of reducing their dominance. ■

Luc Dumontier is head of factor investing and Guillaume Garchery is head of R&D, both at La Française Investment Solutions in Paris.

[1] Universe comprised of common stocks that appeared at least once in the database of the Center for Research in Security Prices between 1926 and 2016.
[2] The risk-free rate is represented by one-month Treasury bills.
[3] The Gafam composite comprises five companies but includes six stocks, as Google is represented by two shares (A and C) of its holding company Alphabet.
[4] Capitalisation-weighted performance with dividends reinvested.
[5] The universe includes the stocks that were part of the S&P at least once between December 2014 and March 2018.
[6] Annualised volatility using daily data over rolling 60-day periods.
[7] Results obtained by regressing the equally weighted returns of Gafams versus those of the S&P over a rolling 60-day period.



The common drivers behind alt risk premia's difficult year

Statistical analysis shows four strategies caused most pain, but funds suffered differently, write Luc Dumontier **and** Guillaume Garchery **of La Française Investment Solutions**

Those who were waiting for the alternative risk premia (ARP) sector to face its first real test got their wish in 2018.

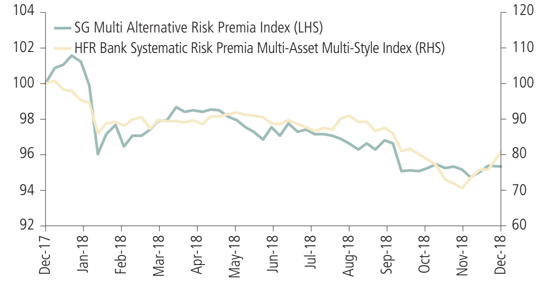
The year was disappointing for the main indexes of both bank products and funds, with performance below the risk-free rate[1] in eight months out of 12 and an average Sharpe ratio of –1.2 (see figures 1 and 2).

What happened? Here we dig into the data to understand which strategies drove losses industry-wide and to look at how individual funds were exposed.

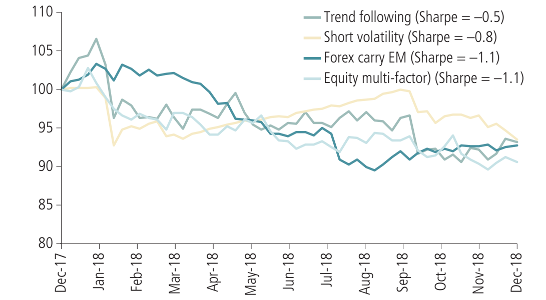
The results show funds were hurt, above all, by losses in four areas: short volatility, trend following, foreign exchange carry and equity multi-factor strategies.

They also show, however, that experience varied across funds as the risks inherent in differences in ARP approaches materialised. These risks included overly simple factor construction and inadvertent market exposure, forced selling/deleveraging, re-correlation risk and lack of diversification.

1. Both bank ARP products and ARP funds suffered in 2018

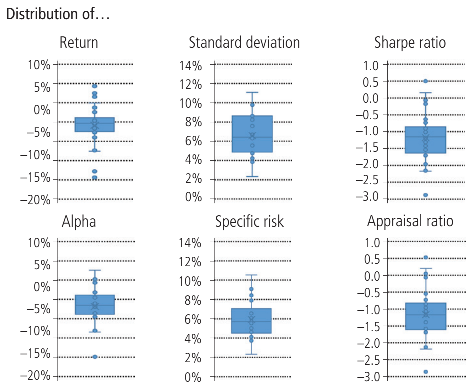


2. Four strategies are in the spotlight as the main cause of last year's performance slump

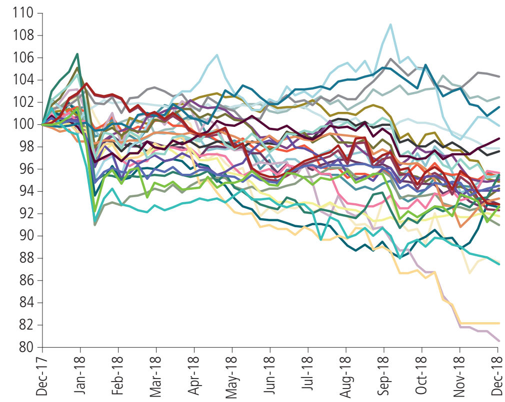


Source: Bloomberg, JP Morgan. Standard deviations/Sharpe ratios are calculated using weekly data from December 29, 2017 to December 31, 2018.
Trend following = HFRX Macro Systematic Diversified CTA Index (excess return versus Fed funds).
Short volatility = SGI Vol Premium US Index. Forex carry EM = DB Emerging Market Currencies Basket Index.
Equity multi-factor = equally weighted basket of value, quality and momentum GDM equity factors from JP Morgan Research.

3. Risk and return metrics among funds were fairly widely distributed...



4. ... but the path of returns suggests these funds were hit by a common risk



Source: Bloomberg – panel of 30 multi-asset/multi-style/long/short mutual funds selected by the authors as being the most representative.
Return measures in US dollar – for funds that only offer share classes in euros, calculations account for spread between Fed funds and Eonia.
Risk measures are calculated using weekly data from December 29, 2017 to December 31, 2018. Alphas and specific risks are calculated versus the S&P 500 Total Return Index.

Figures 3 and 4 show risk and return figures for a group of 30 multi-asset, multi-style funds selected as representative of the ARP industry. The best funds registered slightly positive returns and the worst performers were down by more than 10%. The returns of most funds sat within a band between –4% and –8% for Sharpe ratios between –1 and –1.5.

At the same time, the evolution of the performance of most funds was quite similar over the year. In other words, most seem to have been hit by the realisation of a common risk.

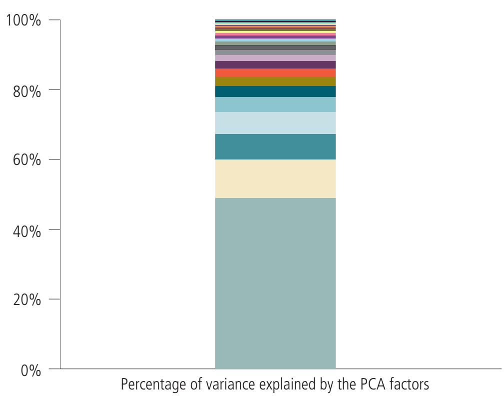
A principal component analysis (PCA) across the 30 funds confirms this visual impression.

Figure 5 shows the percentage of the risk of the funds explained, on average, by the PCA factors. On average, 50% of the risk of the funds is explained by the first factor.

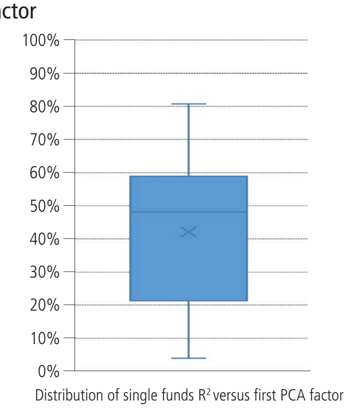
That might seem a small number compared with 90% of risk explained by the first factor for PCA on sub-categories of hedge funds.[2] But the ARP funds displayed highly heterogeneous exposure to the factor. The R^2 or the percentage of risk explained by the funds' exposure to the first factor ranges from 4% to 81% (see figure 6).

At the same time, the cumulative performance of a diversified basket of all funds weighted by their share in the first factor is strikingly similar to that of the global ARP industry (see figure 7). In other words, the first PCA factor almost fully explains the average performance of the ARP industry in 2018.

5. PCA supports this impression...



6. ... though funds varied in their exposure to the first PCA factor

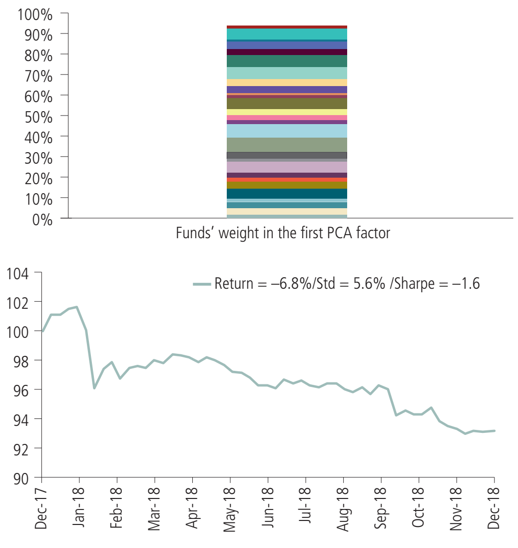


Source: Bloomberg – panel of 30 multi-asset/multi-style/long-short mutual funds selected by the authors as being the most representative.
Return measures in US dollars. For funds that only offer share classes in euros, calculations account for spread between Fed Funds and Eonia.
PCA using weekly data from December 29, 2017 to December 31, 2018.

What does this first PCA factor comprise?

Table A shows the results of six independent regressions, each of which considers a different set of explanatory variables. The first column is the results of the regression of the first PCA factor versus the equity market only, while the last column shows the results of a regression of the same PCA factor versus the four strategies often talked about as the culprits for the ARP sector's poor year (see figure 2).

7. A basket of funds weighted by their share in the first factor shows returns similar to the sector as a whole



Source: Bloomberg – panel of 30 multi-asset/multi-style/long-short mutual funds selected by the authors as being the most representative.
Return measures in US dollars - for funds that only offer share classes in euros, calculations account for spread between Fed Funds and Eonia.
PCA using weekly data from December 29, 2017 to December 31, 2018.

A. The risk of the first PCA factor is almost fully explained by its beta to four strategies

Alpha (weekly)		–0.15%	–0.12%	–0.10%	–0.09%	–0.07%	–0.06%
Beta	Market	0.17**	0.12**	0.04	0.03	0.03	-
	Trend following	-	0.28**	0.20**	0.20**	0.20**	0.19**
	Short volatility	-	-	0.30**	0.29**	0.26**	0.31**
	Forex carry EM	-	-	-	0.14**	0.15**	0.16**
	Equity multi-factor	-	-	-	-	0.13**	0.13**
Adjusted R^2		31%	71%	78%	80%	85%	85%

Note: ** indicates that the variables are significant at 99% level confidence

This last regression has an R^2 of 85%. That is, the risk of the first PCA factor is almost fully explained by its exposure – or beta – to the four selected strategies. Betas are all statistically significant at a 99% level.

The weekly alpha is –0.06%, or approximately –3% annually. This can be attributed to: portfolio management choices, including the different implementation of the selected strategies; dynamic allocation between strategies or additional performance from the 15% of the risk that is unaccounted for; or costs including management fees and transaction costs.

Figure 8 shows the cumulative performance of the replication portfolio of the first factor, allocated in line with the betas of the final regression analysis.

This is calculated as a beta-weighted average of the performance of the four selected strategies, all in excess of cash. To this, the weekly constant of –0.06% – the alpha – is added as well as the performance of the Fed funds rate to simulate a funded solution.

The result is striking. This beta-weighted basket almost perfectly replicates the performance of the first PCA factor with not only the same return, volatility and Sharpe ratio but also the same path of returns.

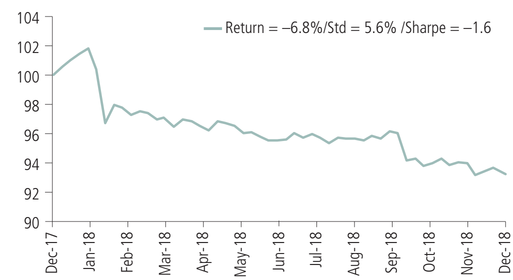
Simply put: the four strategies under the spotlight seem to explain most of the pain felt by ARP funds and products last year.

What about individual funds? A regression analysis of the performance of each of the 30 funds versus the selected strategies yields three major lessons (see table B).

First: how funds implemented the four strategies made a difference. Funds 13 and 23 (in green), for example, are both very representative of the first PCA factor, with almost three-quarters of their total risk explained by their exposure to this factor (the R^2). Moreover, their betas versus the selected strategies are roughly the same.

However, they have a significant difference in alpha – 0.07% on a weekly basis or roughly 3.5% per annum. There could be many explanations for this spread, but the high R^2 results and similarities in beta levels are clues that point towards differences in implementation of the selected strategies, either in terms of design or implementation costs.

8. A replication portfolio also matches the performance of the first factor



Regression and risk measures are calculated by the author using weekly data from December 29, 2017 to December 31, 2018.CTA = HFRX Macro Systematic Diversified CTA Index (excess return versus Fed funds), Short volatility = SGI Vol Premium US. Forex carry EM = DB Emerging Market Currencies Basket Index/ Equity multi-factor = equally weighted basket of value, quality and momentum GDM equity factors from JP Morgan.

Secondly: funds' other strategies made money in some cases, but lost money in others. The managers of funds 6 and 24 (in red) both allocated a significant portion of their risk to additional strategies other than the four identified – R^2 below 50%. While betas are similar, weekly alpha levels are opposite: –0.05% for fund 6 versus +0.04% for fund 24, a spread of 4.5% on a yearly basis.

The explanation that one fund implemented the selected strategies better remains plausible. But another likely hypothesis is that the additional strategies in these funds delivered positive performance for fund 24, but destroyed value for fund 6.

Finally, some funds largely avoided the four strategies altogether. Funds 4 and 11 (in blue) had a very low allocation to the four selected strategies in 2018 (R^2 is minimal), and especially low betas versus the equity multi-factor. Assuming this is a structural decision, can funds that do not implement these most-documented, least-debated premia really be considered ARP funds?

Why did the four strategies in the spotlight suffer?

In an article on *Risk.net* last year, we warned that trend-following strategies that had entered 2018 with very high long exposure to equity markets and the volatility premium were negatively exposed to a sharp decline in the equity markets. These risks materialised – principally in February (see figure 1).

Naively built foreign exchange carry strategies, which are long the highest-yielding currencies and short the lowest-yielding currencies, often suffer in line with unexpected falls in GDP. And, last year, the protectionist stance of the US on trade clearly hurt strategies of this type, especially in emerging markets.

The ARP funds most exposed to equity alternative premia delivered very strong performance in 2017. When things reversed in 2018, these same funds were penalised. The situation was aggravated further at the end of the year by forced selling/deleveraging linked to significant redemptions from long/short equity strategies.[3] A final factor that came into play was re-correlation risk – the Achilles' heel of ARP funds.

There were nevertheless two bright spots for the sector. First, specific risk – a strategy's risk that does not come from exposure to the overall equity market[4] – represented more than 80% of the total risk both for bank indexes and funds. ARP solutions had, on average, limited exposure to the overall equity market. That contrasts with traditional hedge funds, where specific risk represented only a third of their total risk.

Secondly, the ARP industry stood out favourably in December. Societe Generale's ARP index returned +2.9% in a month when the HFRX Global Hedge Fund Index lost 1.9%.

The sector's results in 2018, then, point not so much to problems with factor investing per se, but strengthen the case that the universe of ARP solutions is highly heterogeneous, and investors must choose carefully between the options available. ■

Luc Dumontier is a partner and head of factor investing, and Guillaume Garchery is partner and head of quantitative research and development at La Française Investment Solutions (LFIS) in Paris

[1] Fed funds rate.
[2] The hedge funds industry is represented by the HFRX Global Hedge Fund Index.
[3] According to Wilshire Associates, the US universe of liquid alternative funds declined $31.4 billion in 2018, with dozens of funds liquidated.
[4] The market is represented by the S&P 500 Total Return Index.

B. Experience across funds highlighting differences in implementation

Regression of funds ranked by descending R^2

		Beta				
	Alpha	Trend following	Short volatility	Forex carry EM	Equity multi-factor	Adjusted R^2
Fund 26	-0.05%	0.08	0.46	0.04	0.11	75%
Fund 23	0.01%	0.24	0.61	0.12	0.04	75%
Fund 13	-0.06%	0.25	0.70	0.13	0.08	74%
Fund 18	-0.01%	0.31	0.25	0.23	0.21	71%
Fund 12	-0.03%	0.16	0.30	-0.02	0.02	69%
Fund 30	-0.03%	0.04	0.90	0.18	0.02	61%
Fund 25	-0.03%	0.25	0.11	-0.13	0.05	57%
Fund 28	-0.13%	0.16	0.47	0.33	0.16	54%
Fund 1	0.03%	-0.01	0.24	0.06	0.08	54%
Fund 24	0.04%	0.32	0.31	0.34	0.18	49%
Fund 10	-0.26%	0.23	0.23	0.68	0.15	49%
Fund 7	-0.06%	0.03	0.44	0.13	-0.01	43%
Fund 14	0.08%	0.38	0.42	-0.03	0.07	43%
Fund 17	-0.13%	0.11	0.24	0.11	0.03	43%
Fund 6	-0.05%	0.25	0.23	0.25	0.19	42%
Fund 21	-0.07%	0.15	0.22	0.32	-0.06	42%
Fund 9	-0.13%	0.15	0.11	0.04	0.07	41%
Fund 8	-0.04%	0.09	-0.01	0.28	0.17	39%
Fund 2	-0.18%	0.35	-0.20	0.00	0.44	32%
Fund 22	-0.31%	0.24	-0.05	0.19	0.30	31%
Fund 16	-0.06%	0.20	-0.02	0.00	0.20	27%
Fund 15	-0.09%	0.20	-0.04	-0.03	0.15	25%
Fund 5	-0.11%	0.13	-0.10	-0.33	0.31	23%
Fund 3	-0.04%	0.14	0.03	0.18	0.23	18%
Fund 20	-0.16%	0.12	-0.14	-0.14	0.16	16%
Fund 19	-0.10%	-0.05	0.19	0.05	0.31	15%
Fund 29	-0.09%	-0.04	0.18	0.05	0.31	15%
Fund 4	-0.07%	-0.05	-0.02	0.08	0.00	8%
Fund 11	0.07%	0.13	-0.04	0.13	-0.02	7%
Fund 27	0.02%	-0.05	0.22	-0.08	0.07	5%

>> Further reading

- Bull run shows up differences in how factor strategies are built *www.risk.net/5393526*
- Why factor crowding fears are overblown *www.risk.net/5341166*
- Why re-correlation matters in alternative premia investing *www.risk.net/2473808*

Bull run shows up differences in how factor strategies are built

The wide differences in factor strategies' performance in 2017 reflect market exposure, factor construction and risk budgeting, writes Luc Dumontier **of La Française Investment Solutions**

Last year, the S&P 500 delivered a net total return of about 21% – its best yearly performance since the launch of the first factor-investing strategies. Annualised volatility was around 7%, and implied volatility dipped regularly below 10%.

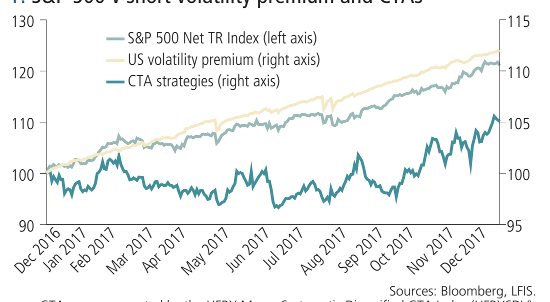
Yet the performance of different factor strategies varied widely – both for strategies based on different premia and for specific implementations of strategies based on the same premia. Why?

A look back at the year shows how the answer lies in market exposures, the construction choices implicit in factor products, and how exposures in multi-factor funds are balanced.

Underlying premia

Firstly, the equity market exposure of different strategies generated much of the performance disparity. Two examples of strategies that benefitted were the volatility premium on the S&P and trend-following strategies such as commodity trading advisers (CTAs).

1. S&P 500 v short volatility premium and CTAs



Sources: Bloomberg, LFIS.
CTAs are represented by the HFRX Macro Systematic Diversified CTA Index (HFRXSDV);
US volatility premium is represented by the SGI Vol Premium US Index (SGIXVPUX)

The volatility strategy on the S&P consists of monetising the difference between implied and realised volatility, for example by rolling one-month variance swaps. This approach generated an excess return of almost 12% in 2017 (see figure 1) with a high correlation with the S&P of 60%, according to the SGI Vol Premium US Index. The realised beta of the strategy was 0.2 in 2017.

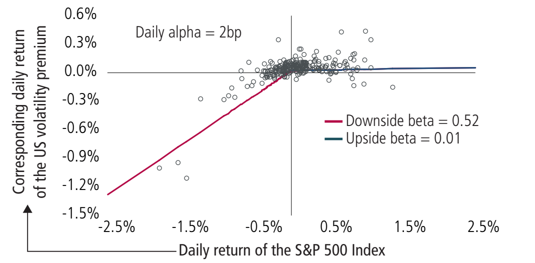
However there is a worrying bias worth noting here. The strategy has strong negative convexity. Estimating the beta of the strategy when the daily returns of the S&P were positive (the blue line in figure 2) and negative (the red line), it looks similar to selling out-of-the money put options: investors accepted a downside beta of more than 0.5 in 2017 to capture daily alpha of just two basis points.

Compare that with 2012, which was also a low volatility period and when the volatility premium strategy on the S&P returned a similarly high 14%. In 2012, however, as figure 3 shows, investors captured twice the amount of alpha (five basis points) with only half the level of downside beta risk (0.31).

Furthermore, 2017's 0.52 downside beta is the result of a historical regression which actually underestimates the current risk of the strategy. An instantaneous measure of the downside risk is the "95–100 downside skew", or the difference between one-month implied volatility levels at strikes of 95% and 100% of the current spot price. This is effectively an estimate of the rise in implied volatility that would follow an instantaneous decline of 5% in equity markets. Figure 4 shows that the current skew is 6.4% and in the extreme range of its historical distribution. That is to say, the volatility premium strategy has rarely been more negatively exposed to a sharp decline in the equity markets. At today's levels, this spring is tightly coiled and the spread could be expected to snap higher, triggering large losses for volatility premia strategies.

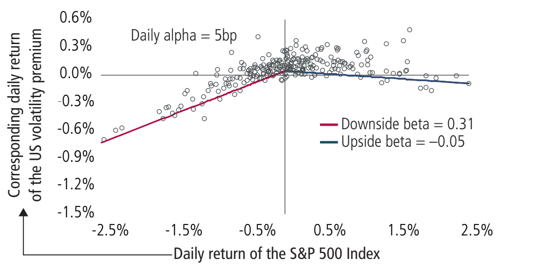
CTA strategies that rely on trends in asset classes also profited from the

2. Risk/return profile of the "short volatility strategy" in 2017



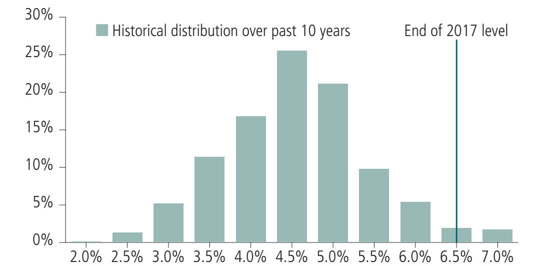
Sources: Bloomberg, LFIS.
US volatility premium is represented by the SGI Vol Premium US Index (SGIXVPUX)

3. Risk/return profile of the "short volatility strategy" in 2012



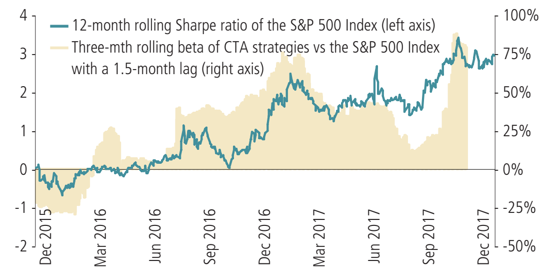
Sources: Bloomberg, LFIS.
US volatility premium is represented by the SGI Vol Premium US Index (SGIXVPUX)

4. S&P 500 95–100% 'negative skew' on one-month maturity



Source: LFIS and Bloomberg

5. Equity market exposure of CTA strategies



Sources: Bloomberg, LFIS.
CTAs are represented by the HFRX Macro Systematic Diversified CTA Index (HFRXSDV)

steady performance of equity markets, especially in the fourth quarter.

A rolling three-month regression of the performance of CTA funds versus annual returns of the S&P shows this.

In figure 5, the line on the left axis corresponds to the one-year Sharpe ratio of the S&P, while the shaded area corresponds to the exposure of CTAs to the S&P estimated by the regression analysis. We have lagged exposure levels by six weeks as the calculated beta corresponds to an average over three months. The result: CTAs had average exposure – implicit or explicit – to the S&P of more than 80% in the fourth quarter of 2017. This explains their excellent performance in the last months of the year (see figure 1).

The one-year Sharpe ratio of the S&P remained very high and steady in the fourth quarter, at around three. CTAs therefore entered 2018 with raised equity exposure. The ratio between the performance of CTAs and the S&P in 2018 year-to-date, as at January 24, is 94% – a YTD return of 5.8% for CTAs compared with 6.2% for the S&P.

Product construction

A second reason for differences in factor strategy performance comes from how strategies are built. Industry specialists broadly attribute differences in implementation to three things: nuances in factor definitions, the mechanics of stock-weighting approaches, and actual portfolio turnover and trading costs. Even for momentum, which at first looks simple because it depends only on stock prices, the implementation choices are considerable.

Questions include, for example, whether to use total return or price return, whether to adjust returns for risk, and which risk measures to use – beta, volatility or idiosyncratic volatility.

More recent returns might be overweighted using exponential averages. Momentum can be measured over different periods, usually 18, 12 or six months. Often the most recent period, in which mean-reverting phenomenon usually appear, is ignored. Sometimes, strategies consider returns from before the formation period.

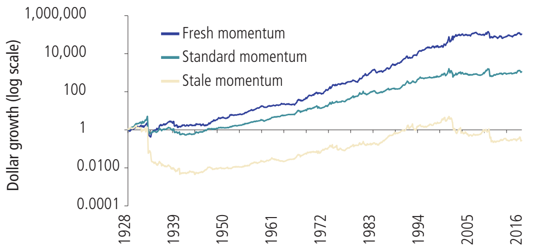
Rob Arnott et al distinguish between "standard," "stale," and "fresh" momentum signals. Standard signals are based on the last 12-month performance, ignoring all other information about prior returns (the green line in figure 6).

The stale momentum 'strategy' selects stocks with the most extreme performance in the same direction used for momentum selection in the 12 months preceding the last year (the yellow line).

A fresh momentum approach selects stocks with the most extreme performance in the opposite direction to that used for momentum selection in the 12 months preceding the last year (blue line). The idea is to avoid buying or selling stocks that are too expensive or cheap. As figure 6 shows, the results are very different from one to another.

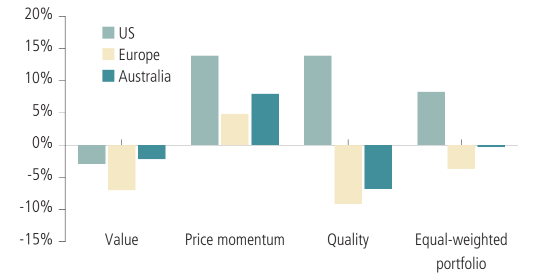
Stock-weighting approaches also offer a number of implementation choices. Once the stocks are selected, should equal weights be favoured over

6. Comparison of fresh, standard and stale momentum long/short US portfolios



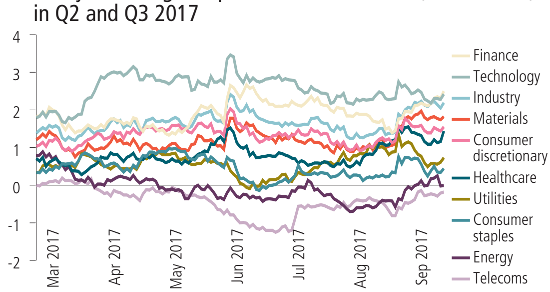
Source: Research Affiliates

8. Equity-style premia by country in 2017



Sources: JP Morgan, LFIS

7. One-year rolling Sharpe ratios of US sectors (GICS level 1) in Q2 and Q3 2017



Sources: Bloomberg, LFIS

a capitalisation approach? Or should weights depend on the strength of the score? How many stocks should be bought and sold? Should you build the long portfolio using index futures, or by also shorting single stocks? Should you risk-adjust the short leg to the long leg? Based on which risk: volatility, beta? Should the allocation be country-neutral? Sector-neutral? Or should both inter- and intra-sector bets be considered?

In 2017, equity momentum strategies where inter-sector bets were permitted benefitted from the gradual outperformance of technology, financial and industrial sectors versus energy, telecom and consumer staples. During the second and third quarters, it is striking how far the ranking of rolling one-year Sharpe ratios for the US sectors (GICS level 1) remained almost unchanged (see figure 7).

Portfolio turnover and trading costs also contribute to performance dispersion. Different strategies rebalance with frequencies from daily to monthly. Some require entry or exit signals to persist over several days to buy or sell stocks.

Strategies employ different approaches to placing market orders to rebalance the portfolio. Controlling turnover and the manager's investment infrastructure are both key. Several studies, including from Novy-Marx and Velikov, show that strategies with low turnover, such as value and size, incur small to moderate trading costs, while higher turnover strategies, like momentum and low-risk, can have trading costs high enough to wipe out the alpha.

Risk budgeting

Thirdly, investment solutions that rely on the same premia and are implemented in the same manner can still deliver different performance if they have different risk budget allocations.

The performance of equity premia designed by the same research team at JP Morgan but in different countries provide a good example (see figure 8).

The value and momentum premia delivered similar performance across different countries. However, quality saw strong performance in the US and delivered negative returns in Europe and Australia. The overall impact for the equal-weighted portfolio was that average total performance was positive for the US, but negative or flat elsewhere.

The big winners of 2017 were of three types: strategies that were explicitly or implicitly exposed to risky asset classes; equity alternative premia, especially those implemented in the US market; and strategies that extended the price momentum approach to inter-sector trades.

Looking back, the experience across the industry reinforces the point that factor investing is an investment framework rather than a standalone strategy. Implementation choices are critical. And for end investors, these strategies should be selected by taking into account other investments already in their portfolios. ■

Luc Dumontier is a partner and head of factor investing at La Française Investment Solutions in Paris. This article was written with contributions from Guillaume Dupin, Guillaume Garchery and Yann Le Her, all partners and senior portfolio managers at LFIS.

>> Further reading

- Mannix R, 2017
  'Great rotation' highlights clash over unseen risks in factor investing
  www.risk.net/2480714
- Arnott R, V Kalesnik, E Kose and L Wu, 2017
  Can momentum investing be saved? *www.researchaffiliates.com/en_us/publications/articles/637-can-momentum-investing-be-saved.html*
- Novy-Marx R and M Velikov, 2014
  A taxonomy of anomalies and their trading costs
  papers.ssrn.com/sol3/papers.cfm?abstract_id=2535173

Commodity premia
It's all about risk control

With numerous strategies that offer attractive rewards for assuming risks that other market participants are unwilling to bear, commodities are well suited to a risk premia approach. However, common commodity premia can feature exposure to unwanted risks. Luc Dumontier **and** Guillaume Garchery **present the main commodity risk premia, ways to implement them and pitfalls to avoid**

The impact of repeated crises – the bursting of the technology bubble in 2001/2002, the global financial crisis of late 2008/early 2009 and the sovereign debt crisis of 2011/2012 – on the performance of traditional asset allocation funds has been amply documented. Professionals have recognised that allocating between traditional (equities and bonds) and alternative (real estate, commodities, infrastructure, private equity, etc) asset classes resulted in an implied exposure to, at best, only a few risk factors in benign conditions and, at worst, to a single equity/liquidity factor in times of strong risk aversion.

Rather than investing in asset classes that tend to re-correlate at the worst time, a new generation of multi-asset products allows for direct investment in the alternative premia that can be extracted from various asset classes. These premia reward investors either for assuming an additional structural risk (economic or financial) that cannot be diversified away, or for their ability to develop strategies that benefit from the behavioural biases and/or regulatory constraints that other market participants face. The best-known premia are those that can be extracted from equities including value and size (Fama French, 1992), momentum (Carhart, 1997), low risk (Haugen and Heins, 1975) and quality (Asness, Frazzini & Pedersen, 2013). These premia can be replicated using long/short portfolios – for example, by purchasing the lowest price-to-book stocks and selling the highest price-to-book stocks to capture the equity value premium.

Premia on other asset classes have recently come into focus, with those on commodities probably among the least documented. Yet the commodity space is particularly well-suited to this thematic, with numerous investable premia that offer attractive rewards for assuming risks that other market participants – including consumers, producers and inflation hedgers – are unwilling to bear on a systematic basis. However, the commodity market is very specific and most common risk premia also feature potential exposure to unwanted risks that may cause strong drawdowns. In this paper, we present the main commodity risk premia, various ways to implement them and the main pitfalls to avoid.

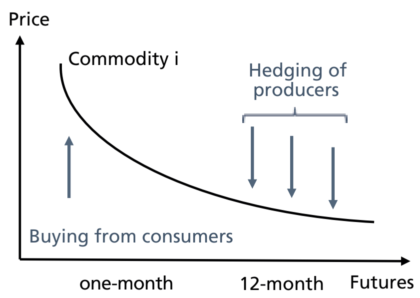
Where do commodity premia come from?

In commodities, the best-known premia are curve, liquidity, trend, carry/value and volatility premia. Commodity curve premia are linked to Keynes's theory of normal backwardation, which states that commodity producers sell long-dated contracts at a discount in order to hedge their output, whereas consumers often buy short-dated contracts at a premium in order to secure near-time consumption. Therefore, investors who buy from producers and sell to consumers can capture an 'insurance risk premium' in the form of the roll yield (see figure 1).

Commodity liquidity premia stem from the congested roll periods of traditional commodity benchmarks. Both the S&P GSCI and Bloomberg Commodity (BCOM) reference indexes track prices of futures contracts on single commodities with predefined monthly contract roll schedules

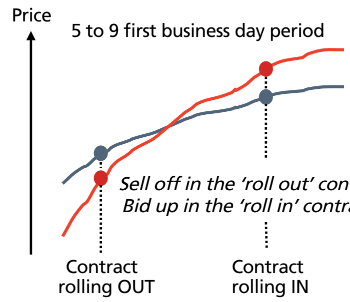
Byjeng / Shutterstock.com

1. Illustration of the curve premia



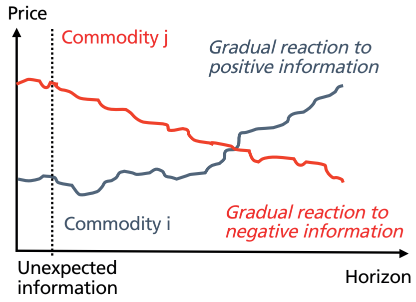
Source: La Française Investment Solutions

2. Illustration of the liquidity premia



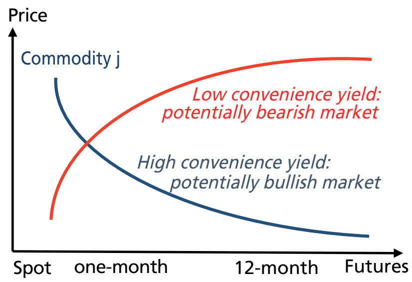
Source: La Française Investment Solutions

3. Illustration of the trend premia



Source: La Française Investment Solutions

4. Illustration of the carry/value premia

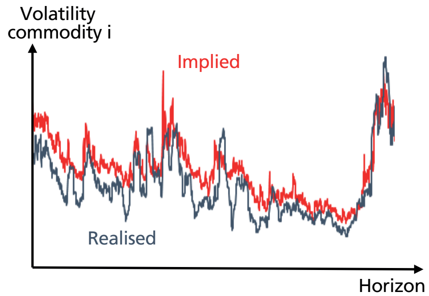


Source: La Française Investment Solutions

(typically between the fifth and the ninth business day of the calendar month). We estimate, on the basis of data provided by the Commodity Futures Trading Commission (CFTC), that approximately one third of the volume traded during the roll period is attributable to the products that track these benchmarks, which tend to weigh on the spread between the old contract and the new one. Investors in liquidity premia take advantage of these structural flows to carry the spread 'new future versus old future' before the congested roll period and the opposite spread after (see figure 2).

As with other asset classes, commodity trend premia seek to benefit from the stylised fact that past winners continue to outperform past losers for some time in the future for reasons that are often behavioural (gradual diffusion of new information, extrapolation and so on). Investors in commodity trend premia allocate between commodities, applying positive weights to those that have recently outperformed and negative weights to those that have underperformed over the same lookback period (see figure 3).

5. Illustration of the volatility premia



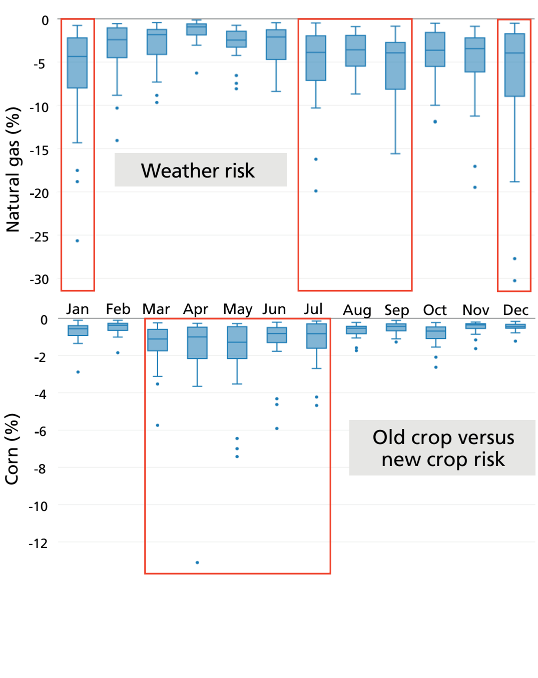
Source: La Française Investment Solutions

The choice of name for the following premia – carry or value – is difficult, as is the case in the equity space. Indeed, the strategy goes long the most backwardated commodities and short the most contangoed commodities to take advantage of both positive carry and timely relative spot appreciation of the long portfolio versus the short portfolio. Backwardated curves may indicate scarcity of supply in the spot market whereas contangoed curves typically indicate an abundance of supply (see figure 4).

Investors in commodity volatility premia aim to capture the structural spread that exists between the implied volatility and the realised volatility of single commodities. In commodity markets, the main driver of the volatility risk premium is consumers and producers using options to hedge their commodity price exposure while there are few natural sellers of optionality who have to be compensated for taking the risk of losses during stressed periods (for example, since the start of 2015, see figure 5).

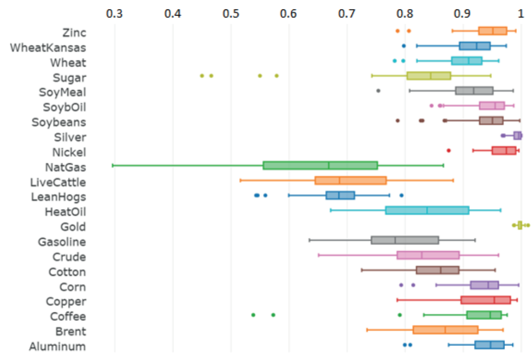
In sum, there are three broad ways to seize commodity premia: (i) intra-curve positioning such as curve and liquidity premia, (ii) inter-commodity premia such as trend and carry/value premia and (iii) classical volatility premia. Each of these three implementation approaches is exposed to several unwanted risks that savvy investors can overcome.

6. Maximum drawdown (F3–F0) time spread



Sources: La Française Investment Solutions, Bloomberg

7. (F3–F0) time spread beta



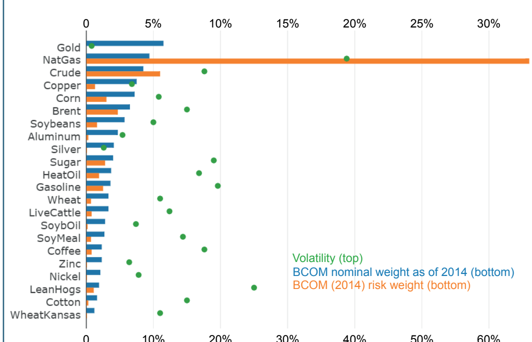
Sources: La Française Investment Solutions, Bloomberg

What are the main risks of intra-curve positioning and how can they be mitigated?

As detailed in the previous section, seizing commodity curve premia may result in carrying the spread between the third nearby contract and the first – namely the (F3–F0) time spread. Analysis of the track record of curve strategies shows that most historical drawdowns were caused by weather-related and seasonal risk factors. Figure 6 shows the drawdowns of (F3–F0) time spreads for two seasonal commodities – natural gas and corn – per calendar month since 1991. Natural gas risks are mostly weather related – linked to demand (hot summers or cold winters) and supply (hurricanes). Corn risks generally materialise during the US planting season (between spring and the beginning of summer), when inventories are low and the crop is very sensitive to weather events.

Due to the seasonal nature of some commodities, these risks can be predicted and mitigated. In practice, investors simply have to carry the

8. (F3–F0) time spread risk



Sources: La Française Investment Solutions, Bloomberg spread of contracts that are sensitive to the same seasonal effects. For example for corn, rather that carrying the 'September versus May' spread (as most customised indices do), investors should instead carry the 'July versus May' spread.

The second main pitfall when implementing commodity premia related to curve positioning comes from the fact that the risk of different contracts on the same commodity may be very different. As an illustration, figure 7 displays the beta of the third nearby contract versus the first for various commodities. Front-month contracts are generally more risky as they are more exposed to short-term supply and demand effects. The lowest beta is approximately 0.7, observed for lean hogs, live cattle and natural gas.

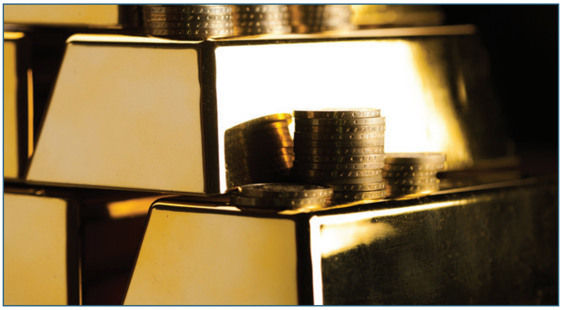
In practice, this means that if an investor purchases a nominal of $100 on F3 and sells a nominal of $100 on F0, the spread has a residual beta of –0.3 or a short position of $30 to the underlying commodity. To build a commodity-neutral spread, the nominal of the short and the long legs should be adjusted depending on their relative beta.

The third possible pitfall comes from the fact that the (F3–F0) time spread of certain commodities is much more volatile (for example, 20% for natural gas) than that of other commodities (less than 5% for metals). However, most investors implement the strategy using the nominal weights of the BCOM index, resulting in an overweight to natural gas (10% in terms of nominals) – the commodity with the most volatile spread. Consequently, 65% of the risk (in variance terms) of their strategy is linked to this spread (see figure 8).

To build a more robust strategy, one approach is to implement nominal weights that depend on the relative risk of spreads (for example, allocation using an equal risk contribution methodology).

What are the main risks of inter-commodity allocation and how can they be mitigated?

The strength of the risk premia approach lies primarily in the capacity to combine numerous and uncorrelated premia rather than optimising the implementation of each premia. This is why most risk premia managers seek to avoid errors relative to valuation and risk models as much as possible. Thus, the criteria for assessing the attractiveness of the assets are very intuitive (for example, price-to-book ratio for equity value premia). Similarly, portfolio construction rules are usually simple (for example, equal long positions on the 10% of stocks that display the lowest price-to-book value (PBV) and equal short positions on the 10% of stocks that display the highest PBV, the nominal of overall long and short portfolios being equal).



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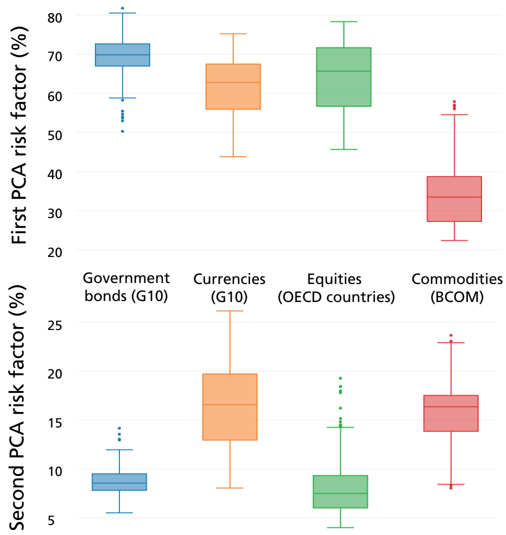
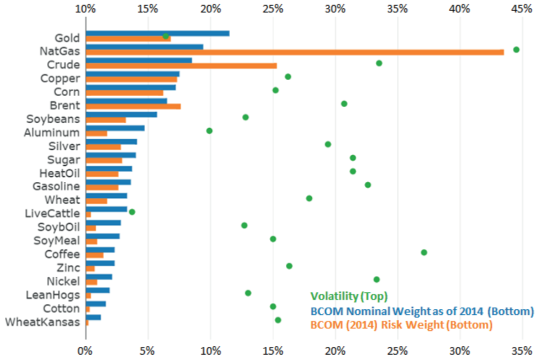
9. PCA within the major asset classes



Sources: La Française Investment Solutions, Bloomberg

10. BCOM index weights in nominal and risk terms



Sources: La Française Investment Solutions, Bloomberg

One may feel that these portfolio construction principles are too simplistic for the commodity space. Indeed, comparing gold, lean hogs and natural gas is not straightforward, as the determinants of their returns are certainly very different. This suspicion is confirmed by the data. Figure 9 illustrates the results of four principal component analyses (PCA) carried out within the four major asset classes, using one-year rolling weekly data since 2001. For nominal government bonds (NGB), the analysis is done using the 10-year yields of the G10 countries. The same G10 universe is used for currencies (FX). For equities (EQY), we run the PCA between the MSCI indices of the 23 countries of the OECD developed universe. Finally for commodities (CMDY), we use the front-month futures of the 22 constituents of the BCOM index.

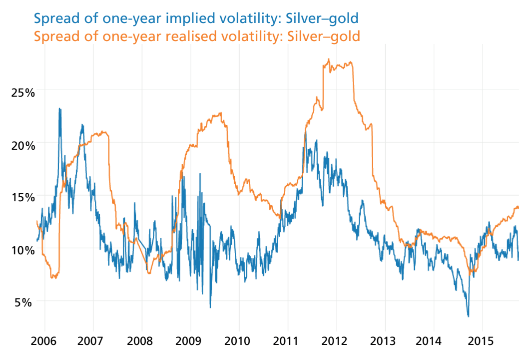
For bonds, currencies and equities, between 65% and 70% of the variance of the assets is, on average, explained by the first factor – in other words, the market factor. For commodities, this is much less the case: on average, only 30% of the variance of single commodities is explained by the whole commodity factor. On the contrary, the second factor tends to be more significant for commodities than for the other asset classes with the exception of currencies. This shows that the commodity universe has globally low levels of correlation – significantly below that of other asset classes – but with subgroups of commodities that are highly correlated. Furthermore, figure 10 shows that the volatilities of various commodities are strongly heterogeneous, from 17% for aluminum to 45% for natural gas.

The main consequence of these findings is that most investors compare apples and oranges in the commodity space without necessarily realising it. For example, most popular carry strategies simply sell the five most contangoed commodities and buy the five most backwardated commodities. Considering the slope of the front-month contract versus the one-year forward contract, a current portfolio based on this approach (as of end of September 2015) would carry short positions on natural gas, Brent, WTI, gas oil and heating oil, and long positions on lean hogs, soybean meal, cocoa, gold and copper. This might have been a good idea over the past few months, but investors should note that this portfolio is highly concentrated in a strong bet on energy commodities (short) versus others (long). More generally, there is a significant chance that the portfolio is not immune to developments in the commodity market as a whole and potentially bears unwanted risks that may result in strong drawdowns. If simplicity is often preferred for portfolio construction, inter-commodity premia require the intervention of a covariance matrix, or at least the implementation of sector-neutral positions.

What are the main risks of commodity volatility strategies and how can they be mitigated?

According to Pimco, commodity markets historically exhibit one of the highest volatility risk premia of all asset classes. Systematic short volatility strategies were initially implemented in oil-related commodities for liquidity reasons. Unfortunately, as indicated in figure 11 (analysis performed by JP Morgan between September 2010 and September 2015), this kind of strategy has displayed a correlation of close to 30% over the past five years to both the same strategy in the equity space and the equity market itself.

12. Volatility spread silver versus gold



Sources: La Française Investment Solutions, Bloomberg

11. Correlation of short volatility strategies

Source: JP Morgan

	WTI	Brent	Heating oil	Gold	Corn	Soybeans	Wheat	Sugar	Cocoa	Coffee	Live cattle	Equity volatility	MSCI World
WTI	100												
Brent	89	100											
Heating oil	83	88	100										
Gold	26	21	21	100									
Corn	22	19	20	17	100								
Soybeans	17	17	19	14	50	100							
Wheat	21	20	22	17	65	41	100						
Sugar	17	14	13	13	15	15	14	100					
Cocoa	6	2	5	2	2	3	-1	2	100				
Coffee	8	5	3	6	9	15	9	16	8	100			
Live cattle	7	3	2	11	14	11	7	9	3	4	100		
Equity volatility	26	26	26	34	4	6	11	-1	3	9	10	100	
MSCI World	31	29	25	34	19	18	22	9	6	8	16	42	100

This issue may be mitigated by diversifying across other commodities. Indeed, short volatility strategies implemented for sugar, cocoa and coffee show very low correlation to other commodities, even during stressed periods (see figure 11).

Nevertheless, the best way to hedge the strategy against strong changes in overall implied volatility is to consider the volatility space as an asset class in its own right and build long/short portfolios. As an example, figure 12 compares the one-year implied volatility of the spread of silver versus gold and the one-year realised volatility of the same spread. This relative value position is very attractive for at least two reasons. First, there seems to be a premium to the extent that the realised spread is almost always above the implied spread. This finding is all the more appealing as we can clearly explain its rationale: hedging activity is much stronger in the gold space than for silver. Secondly, the implied spread doesn't display trends in stressed periods (for example, 2008), meaning that this position provides returns that are not linked to the overall market context. The commodity space is full of such relative value premia, which offer the potential to be profitable, understandable and lowly correlated.

Conclusion

This paper provides some recipes to limit, to the extent possible, unwanted risks to which the most common risk premia are exposed. Readers might wonder about the impact of these improvements on the risk-return profile of commodities. The short and direct answer is that the analysis of simulated and actual (since end of 2013) track records shows that the improvements can help to almost double the Sharpe ratio of the various commodity premia strategies. The observations in this paper are based on long-term historical simulations, which necessarily have their limitations. Nevertheless, the results provide valuable insight into the functioning of commodities markets and point to areas to focus on when implementing commodity premia. ■

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Overcrowding overstated?

'Overcrowding' is the new buzzword in the factor investing community. Luc Dumontier **and** Guillaume Garchery **from La Française Investment Solutions explain why this fear is largely overstated**

The rush into factor investing strategies – today worth about $1 trillion – has raised concerns that alternative factors are becoming crowded, and therefore overpriced and more sensitive to dislocation events. Is this fear justified?

This article seeks to answer that question in relation to equity alternative premia, the best known and most popular premia. The arguments put forward are equally valid for premia in other asset classes.

We first return to fundamentals that underlie the existence of the alternative premia to assess the potential for popularity to impact returns. We then look at how factor exposures have changed across the market. Lastly, we turn to the risks associated with concentrated rather than widespread groups of investors holding exposure to factors.

Persistence depends on whether premia stem from rational expectations or mispricing

The link between possible overcrowding and a decline in performance is not necessarily valid for every factor.

Risk premia

At one end of the spectrum, risk premia remunerate investors for exposure to systematic risk factors that cannot be diversified away. The best example is the equity risk premium, which rewards investors for bearing the risk of an unexpected economic downturn that could translate into a drop in companies' earnings.

Likewise, the most convincing explanations for the historical over-performance of value and small capitalisation stocks are risk-based. Stocks with attractive valuations – based on price-earnings and price-to-book ratios – are vulnerable to share price falls if the reasons for their low valuation intensify: the 'value trap'. Small cap stocks tend to have, on average, more concentrated revenue streams both geographically and in terms of business mix.

Rational investors are unlikely to stop requiring a premium to accept such risks. "Even if an opportunity [resulting from an additional risk] is widely publicized, investors will not change their portfolio decisions, and the relatively high average return will remain," finds Cochrane (1999). On the other hand, if all stocks in the investment universe had comparable valuation multiples (or comparable market capitalisation levels respectively), rational investors would choose not to implement the value factor (or the size factor, respectively). Moreover, it's worth noticing that most of the risk factors – such as value and size – have built-in protection against overvaluation. As stocks become more expensive and larger, they automatically drop out of the relevant investment universe.

Style premia

At the other end of the spectrum, so-called style premia remunerate investors for their capacity (eg, in terms of investment infrastructure, available cash and regulation) to implement strategies that profit from structural biases linked to market participants' behaviour, investment constraints and structural flows. Arbitrage strategies that exploit pricing inefficiencies in the cash (or spot) and futures markets for the same asset fall into this category. This type of opportunity is often due to the inability of market participants to hold the underlying asset, either due to capital requirements or regulatory constraints.

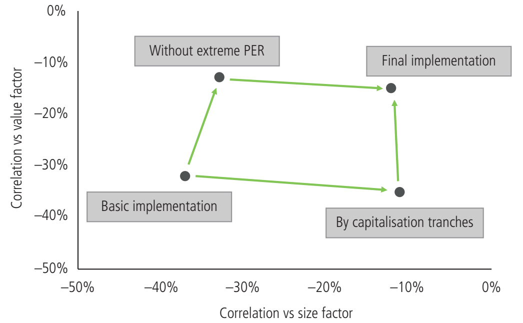
Style premia can be likened to a cake to be shared. The more guests there are, the more rapidly the opportunity will disappear. Mclean and Pontiff (2016) summarised their research into the persistence of style premia thus: "If return predictability reflects mispricing and publication leads sophisticated investors to learn about and trade against the mispricing, then we expect the returns associated with a predictor should disappear. [But there are] frictions [that] prevent arbitrage from fully eliminating mispricing, [such as] transaction costs." Moreover, these arbitrage opportunities are only visible to investors whose scope of counterparty relationships allows them to see the opportunity in the first place – for example, a bank that needs to recycle a given risk. They are mostly accessible via over-the-counter products, which investors need to be able to price and book. And the infrastructure to do that creates a high barrier to entry (see table A).

A. Expected sensitivity of alternative premia to asset raising

		Rationale	Example	Expected sensitivity to asset raising
Alternative premia	Risk premia	**Risk sharing**	Value	**Little**
		Compensation for bearing additional risks		No reason for rational investors to accept risk without return
	Style premia	**Structural constraints**	Cash and carry arbitrage	**Moderate to significant**
		Compensation for having fewer constraints		Sophisticated investors learn about the mispricing
		Information processing	Pair trading	Only those that benefit from a strong investment infrastructure can arbitrage them away and find others
		Compensation for processing information better		

Source: La Française Investment Solutions

1. Different implementations of the low-risk equity strategy



Monthly data from January 2011 to December 2015; Source: Dumontier (see refs)

In-between premia

Finally, there are premia with characteristics of both risk and style premia, such as momentum and low risk. The rationale behind these premia includes both additional systematic risk and investors' behaviour or constraints. Momentum premia are often explained by the anchoring bias, investors' tendency to react only gradually to new information. Since momentum factors are also exposed to sudden reversals, rational investors require premia to hold them. Similarly, investors tend to overpay for riskier assets due to behavioural biases (eg, lottery effect) and investors' constraints (eg, the preference of insurance companies for high-beta stocks in an effort to get more bang for a given regulatory capital charge). But the low risk equity factor tends to realise negative returns when funding liquidity constraints tighten and/or when funding liquidity risk is high, so that it can also be considered a risk factor.

The overcrowding debate is the most heated around these "in-between" premia that do not have a mechanical valuation anchor, ie, inherent overvaluation protection. Some in the industry, most notably Rob Arnott of Research Affiliates, think their valuation multiples are currently high. Meanwhile, other experts such as AQR's Cliff Asness contend that multiples are reasonable by historical standards.

How can we explain this lack of consensus? What has happened is the debate has turned from the factor itself to its practical implementation. For example, the low risk equity strategy is typically implemented by building an equally weighted portfolio that is long the 20% of least-volatile stocks and short the 20% of most-volatile stocks (the long leg being leveraged so the portfolio is globally market-neutral). But Dumontier (2016) shows this 'basic implementation' is negatively correlated to both the value and the size factors, or, in other words, structurally expensive in terms of valuation multiples and capitalisation criteria. This is easily remedied by removing the most expensive and cheapest stocks from the investment universe and combining low-risk portfolios constructed within several tranches of stock capitalisation (see figure 1).

Funds have been exposed to alternative factors for decades

Factor-based assets under management are still dwarfed 20:1 by the market as a whole. Meanwhile, most of the money flooding into the sector is switching from funds that also tilted towards factors in the past – though perhaps less explicitly.

Academic research by Robert Haugen and James Heins highlighted the low risk factor as far back as 1975, and Eugene Fama and Kenneth French documented the value and size factors in 1992. Since then – intentionally or not, both quantitative and fundamental fund managers have skewed their portfolios towards factors to outperform their cap-weighted benchmarks.

The most famous example is Berkshire Hathaway's Warren Buffett, whose performance can largely be explained by exposures to the value, low-risk and quality factors, together with a leverage of about 1.6 to 1 (Frazzini, 2013).

In Carhart (1997) we see persistence in mutual fund performance for a range of US funds over a period of 30 years to 1993 failed to reflect stock-picking skill. "Common factors in stock returns […] explained almost all of the predictability in mutual fund returns," he wrote. Bender et al (2014) showed the same phenomenon in a more recent study (see table B), finding a handful of risk premia indexes accounted for as much as 80% of alpha in US equity markets from 2002 to 2012.

This finding repeats in long-short portfolios. Dumontier (2016) showed during the market dislocation in August 2007 – the so-called 'quant crisis' – equity market neutral funds (as represented by the HFRX sub-index) and equity alternative premia posted significant losses at the same time between August 6–9. This proves that the criteria used by fund managers to select stocks were on average the same as those used to build alternative premia. In Harvey (2016), we find the performance of equity hedge funds from 1996 to 2014, whether systematic or discretionary, was mainly attributable to their exposure to a standard set of factors.

B. Regressions with and without alternative factors

Average across managers (US equity long-only)	Market	Market, value, low risk	Market, value, low risk, momentum	Market, value, momentum, size
Alpha	0.181%	0.060%	0.053%	0.030%
Beta				
Market	1.08	1.15	1.14	0.98
Value		–0.42	–0.20	–0.20
Low risk		0.55	0.55	
Momentum			0.22	0.15
Small cap				0.51
Adjusted R^2	0.86	0.87	0.88	0.92

Monthly data from June 2003 to March 2012; Source: Bender et al. (see refs)

Aggregate exposure to alternative factors is quite limited

One may argue the exposure of 'factor investing' strategies to alternative factors is higher than that of 'active' funds, so that the previous discussion may be incomplete. Blitz (2017) conducted a study on US equity ETFs – the universe where there has been the greatest growth in interest for factor investing strategies. Blitz regressed the returns of 415 US equity ETFs with combined assets under management of more than $1.2 trillion on the returns of size, value, momentum and low-volatility factors over the 2011–2015 period. He split ETFs into those explicitly targeting alternative factors or using alternative weighting formulas versus others he classified as "conventional".

The study showed – as you would expect – that smart beta ETFs were on average positively exposed to alternative factors (see table C). But conventional ETFs showed negative average exposure towards the same factors. At the industry level, the two effects largely cancel each other out.

This occurs because conventional ETFs are often thematic or sector-focused. Funds focused on the biotechnology sector, for example, are negatively exposed to the value factor and those focused on the information technology sector are negatively exposed to the low-risk factor.

C. Aggregate factor exposures of US equities ETFs

Assets weighted aggregate exposure	All ETFs	Smart beta ETFs	Conventional ETFs
Alpha	0.02%	–0.03%	0.04%
Market	0.97	0.97	0.97
Value	–0.03	0.08	–0.08
Low risk	–0.00	0.06	–0.03
Momentum	0.01	0.03	0.01
Small cap	0.03	0.25	–0.06

Monthly data from January 2011 to December 2015; Source: Blitz (see refs)

Concentration can lead to dislocation

That said, while overcrowding might be less of a concern, investor concentration should not be. Overcrowding in terms of the type of investors holding specific factors can fuel dislocation phenomena.

If Blitz's 2017 study were conducted on the global equity universe, the overall exposure to the market factor would be exactly one, with no exposure to alternative factors. Every security has a holder. So, if stocks that embed specific factors are held by only a few investors, they are not held by others.

This highlights a more pernicious danger beyond the potential issue of factor compression, the risk of investor concentration at the level of specific factors. If only a few investors hold most of the assets or if these investors all share common characteristics, such as margin leverage or restrictive liquidity requirements, dislocation events become more likely. This was seen in the 2007 'quant crisis', which occurred even though there is a general consensus that factors were not previously overvalued.

Bayraktar et al (2015) indicate useful directions to gauge the investor concentration for the specific case of the momentum factor, but this field of study is at its inception.

One natural way to mitigate the effects of concentration and potential deleveraging is to diversify a 'factor investing' approach to include other asset classes that are less broadly popular than equity factors.

Ultimately, alternative factors are no more overcrowded today than in the past. The link between overcrowding and a decline in performance is not necessarily valid for every factor. And professionals are far from reaching a consensus on whether or not established alternative factors are overvalued.

Questions surrounding overcrowding are often used to justify somewhat disappointing results and divert attention away from very real issues of data mining, overfitting, cost-ineffective implementation and re-correlation phenomena. ■

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Why re-correlation matters in alternative premia investing

Understanding this key risk can be the difference between success and failure, writes Luc Dumontier

This article studies the conditions in which alternative premia solutions can generate consistent positive returns as well as those that lead to significant drawdowns. A second article will set out 10 commandments to address the risks identified.

If you believe the many simulations by asset managers and investment banks, alternative premia[1] solutions should have delivered regular returns, uncorrelated with traditional asset classes and largely independent of the portfolio construction criteria used.

Since the launch of the first alternative premia funds in 2013, the reality has been quite different.

These strategies have delivered modest results on average, often correlated on the downside with risky asset classes and highly heterogeneous, from one product to another.

It seems the alternative premia label refers more to an analysis framework than a standalone investment strategy. And its robustness depends on choices made in implementation – particularly about correlation risk.

Whether you are a provider or a user of alternative premia solutions, trusting in simulations based on historical data is not enough.

Image: Shutterstock

Correlation

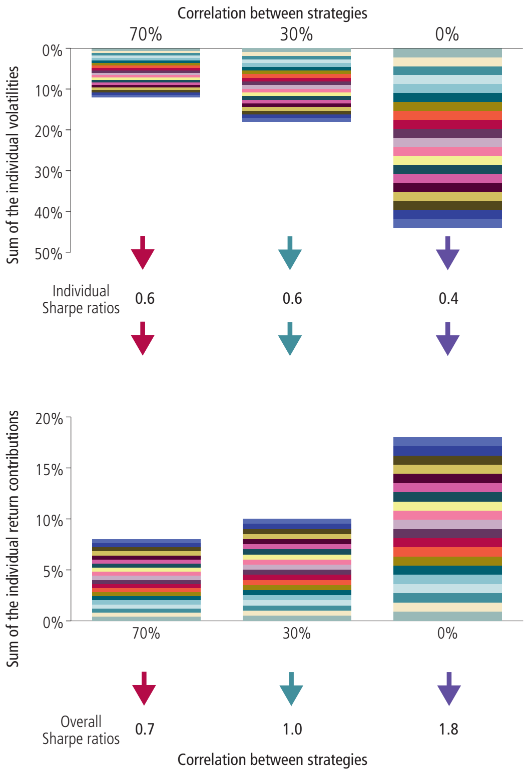
Put simply, success in alternative premia investing depends on the ability to combine uncorrelated strategies.

The classical alternative premia approach combines different long/short portfolios capturing the standard investment styles such as value, carry, momentum, low risk, or liquidity within a broader allocation to traditional asset classes.

These strategies are expected to deliver returns either as remuneration for exposure to an additional risk factor, economic or financial, that cannot be diversified away – often called risk premia – or stemming from biases linked to market participants' behavior, investment constraints and structural flows - often called style premia. Because the rationale behind each individual alternative premia[2] is different, they are expected to deliver largely uncorrelated performance.

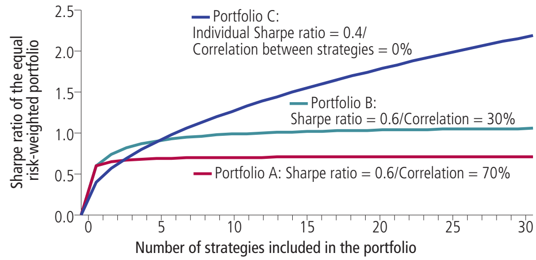
It turns out the level of correlation between the elements of any portfolio has a decisive impact on its risk-adjusted performance. To show this, let's consider a portfolio of 20 strategies[3] that is equally risk-weighted and has an overall target volatility of 10%.

1. Diversification in a portfolio of 20 strategies with a target volatility of 10%



Source: La Française Investment Solutions

2. The power of diversification



Source: La Française Investment Solutions

Figure 1 shows this portfolio can allocate a volatility budget of 0.6% to each strategy if they are 70% correlated, the sum of their individual volatilities being 11.8%; 0.9% if they are 30% correlated, sum of 17.3%; and 2.2% if they are uncorrelated, sum of 44.7%.

The contribution of each strategy to the portfolio's overall performance is simply its volatility budget multiplied by its Sharpe ratio. In this example, we apply a Sharpe ratio that is 50% higher for correlated strategies than for uncorrelated strategies (0.6 vs. 0.4). Notwithstanding, the overall return of the portfolio turns out to be much lower if the 20 strategies are correlated – 7.1% and 10.4%, if the pair-wise correlation is 70% and 30%, respectively – than if they are uncorrelated (17.9%).

So, the portfolios that combine correlated strategies have much lower overall Sharpe ratios (0.7 and 1 respectively) than the portfolio of uncorrelated strategies (1.8). To reach a Sharpe ratio of 1.8 by combining 20 strategies that are 70% correlated (or 30%, respectively), each strategy would have to deliver a Sharpe ratio of 1.52 (or 1.04, respectively) – a highly unlikely scenario.

Figure 2 generalises the study to N strategies (x-axis) that have the same characteristics as previously in terms of pair-wise correlation and individual Sharpe ratio. Portfolio A represents a traditional allocation to risky asset classes, such as equities, corporate bonds and private equity. These have delivered Sharpe ratios in the range of 0.6 over the long term, with correlations among portfolio components averaging 70% or higher in stressed periods. In this case, the overall Sharpe ratio (y-axis) tends towards 0.72 (ie $0.6/\sqrt{70\%}$) – and this limit is approached very rapidly – the Sharpe ratio is already at 0.67 with only three components.

Portfolio B represents traditional multi-strategy hedge funds. These combine individual strategies with target Sharpe ratios of 0.6 and correlations of roughly 30%. In this case, the diversification power is only marginally higher: the overall Sharpe ratio tends towards 1.10 (ie $0.6/\sqrt{30\%}$), a level that is again approached rapidly – the Sharpe ratio is already 0.9 with only five strategies.

Portfolio C represents the stated objective of alternative premia solutions, i.e. to combine many uncorrelated premia, even if they have lower individual Sharpe ratios (0.4 vs 0.6). In this case, the overall Sharpe ratio is potentially unlimited and it is highly profitable to add a new premia, even to an already large portfolio.

The primary lesson is that a portfolio's Sharpe ratio is more dependent on the number of strategies it combines - and especially on the correlation of each strategy to the others – than on each strategy's standalone Sharpe ratio. With Sharpe ratios ranging between 1.5 and 2 based on data over the past 10 to 20 years, simulations of portfolios combining 15 to 20 premia[4] are consistent with these theoretical figures.

But how do we then explain the disappointing returns of most alternative premia solutions since their launch?

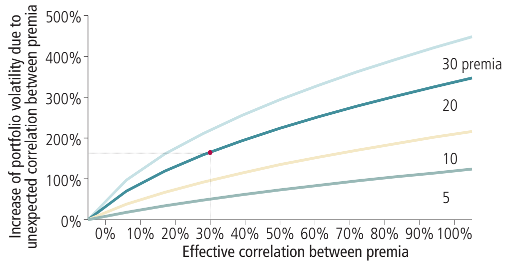
Re-correlation

A big part of the answer is re-correlation. Let's reconsider the equally risk-weighted portfolio of 20 uncorrelated premia and target overall volatility of 10% (see figure 1). As a reminder, the sum of the individual volatilities of the premia in this portfolio is 44.7%.

If in practice, the premia display 30% pair-wise correlations, the overall actual portfolio volatility is 25.9%, an error of nearly 160% versus the initial calibration of 10%. Figure 3 shows the potential calibration error of a portfolio's volatility (y-axis) increases with the number of premia (the different curves), and soars should these premia – initially expected to be uncorrelated – re-correlate strongly (x-axis).

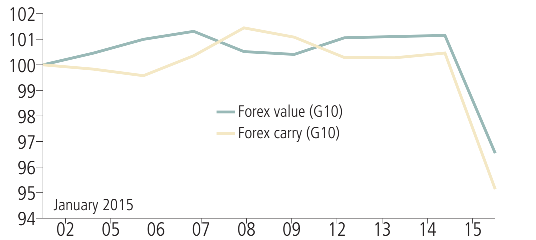
The investor, expecting to benefit from a high level of diversification from holding many uncorrelated premia, is ultimately exposed to a far higher

3. The danger of re-correlation



Source: La Française Investment Solutions

5. Forex carry versus forex value premia in January 2015



Sources: La Française Investment Solutions, Deutsche Bank and Bloomberg

level of risk than desired. The calibration error can be even higher given that re-correlation often occurs in a context of rising volatility across asset classes and thus across premia.

The danger is that premia register negative returns precisely when the portfolio's volatility is exceptionally high resulting in heavy losses. Re-correlation risk is the Achilles' heel of alternative premia strategies. A sound grasp of the circumstances in which this phenomenon can occur is essential to mitigate this risk.

Structural exposure to the same systematic risk

Some alternative premia may be sensitive to a common systematic risk. For example, concerns over global economic growth tend to penalise both high-yielding currencies and value stocks. Currencies that offer an attractive carry are often those of countries whose economies are the most open, cyclical, and/or dependent on commodity exports, such as Australia and New Zealand among G10 countries. Similarly, stocks trading at attractive valuations are most vulnerable to the so-called value trap phenomenon, should reasons for the stocks' low valuation multiples intensify. As an example, forex carry and equity value premia re-correlated on the downside in 2015 (see figure 4), amid mounting concerns about economic growth in the US and China.

Exposure to the same idiosyncratic risk

Different types of alternative premia – for instance carry, value and momentum – can be implemented within the same asset class. These are therefore prone to exposure – with the same directionality – to the same underlying assets. This is even more the case as the investment universe is restricted. If different premia are locally exposed to an asset that performs abnormally, they can display correlated performance.

One of the most spectacular events of 2015 occurred when the Swiss National Bank abandoned its cap on the Swiss franc against the euro. The day of the announcement (January 15), the Swiss franc appreciated by more than 20% against the other G10 currencies, on average. At that time however, the Swiss franc was the least attractive currency in the G10 universe in carry terms (three-month interest rate), and in valuation terms (OECD purchasing power parity), while its price momentum (return over the past 12 months) was among the worst.

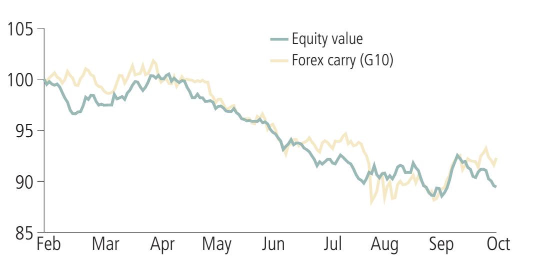
An investor that had overlaid carry, value and momentum premia in the currency universe would likely have accumulated three short positions in the Swiss franc, with harsh consequences in terms of correlation and performance (see figure 5).

Re-correlation on the downside to the underlying asset class

Other alternative premia, initially designed to be insensitive to the underlying market, can end up exposed to it – positively or negatively – during volatile periods.

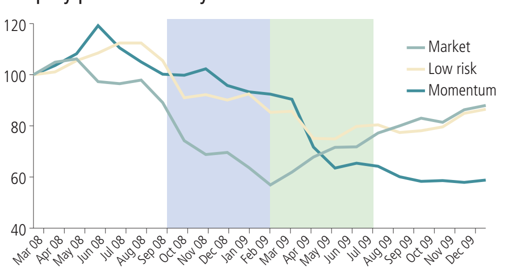
One of the best examples of downward re-correlation is the low risk equity premia during the market collapse of 2008. This premia is usually captured

4. Forex carry versus equity value premia in 2015



Sources: La Française Investment Solutions, Deutsche Bank and Bloomberg

6. Equity premia in the years 2008 and 2009



Sources: La Française Investment Solutions and JP Morgan by building a portfolio that buys less-risky stocks, and sells the riskiest ones. To ensure the portfolio is beta neutral, the long leg is usually leveraged based on the historical beta of stocks, so the portfolio is net long in nominal terms.

However, investors often indiscriminately liquidate all their stock holdings when there is a sharp increase in risk aversion and/or funding liquidity risk. As a consequence, the actual betas of individual stocks converge and the low risk premia may exhibit positive beta versus the market at the worst possible time (see the blue area in figure 6).

If periods of beta compression can be painful for investors, the opposite phenomena of beta decompression can be even more dramatic. The best-known examples are the equity momentum crashes (see Daniel and Moskowitz, 2012), like the one during the market rebound in the second and third quarters of 2009 (see the green area in figure 6) after sharp decreases in previous quarters.

When the market falls significantly over the momentum formation period, assets that fall more than the market tend to be – or become – high beta assets while those that fall less tend to be – or become – low beta assets. Thus, in periods of market decline, momentum portfolios are likely to become long low-beta assets and short high-beta assets, and carry implicit negative exposure to the underlying asset class. If the market rebounds strongly, as was the case for equities in mid-March 2009, momentum strategies can lose the profits of several years in a matter of weeks.

Forced sales/de-leveraging

Co-ordinated forced selling or deleveraging of similarly constructed portfolios can also cause re-correlation of alternative premia. The best-known example is the market dislocation experienced by long/short equity strategies in August 2007 – the so-called 'quant crisis'.

On average, equity market neutral funds (as represented by the HFRX sub-index) lost 5.2% between August 6–9, while equity markets posted only modest declines (-0.9% for the S&P 500 TR Index). This loss corresponds to about 1.5 times the historical annual volatility of the HFRX sub-index (3.5% over the previous three years) and 13 times its three-day volatility (0.4%).

This startling event, which normally (in the statistical sense) has a near-zero probability of occurring, has given rise to numerous studies. Khandani and Lo (2008) explain the losses incurred on the first day as resulting from an initial wave of forced selling by multi-strategy funds or proprietary traders, itself the result of a tightening in liquidity conditions – notably in the aftermath of the liquidation of Bear Stearns' credit funds. This first drop caused many funds to reach their stop-loss limits, thereby aggravating the de-leveraging phenomenon over the following two days. If all equity hedge funds lost ground at the same time, at least initially they must have held the same positions. Analysis of the returns of equity premia over this specific period gives a good indication of what those positions were (see figure 7).

7. Equity premia during the quant crisis

Style	Premia	Sector neutral		Non-sector neutral	
		Loss (%)	Loss (σ)	Loss (%)	Loss (σ)
Value	1-year forward earnings yield	-3.8%	-4.8	-2.2%	-1.9
Value	Free cashflow yield	-3.6%	-5.4	-4.2%	-5.6
Value	Piotroski score	-1.7%	-3.2	-1.7%	-2.9
Quality	Historical ROE	-1.4%	-2.1	-1.5%	-1.8
Quality	Altman Z-Score	-1.7%	-2.7	-1.7%	-2.2
Quality	Open margin 1-year growth	-3.0%	-4.4	-1.9%	-2.5
Momentum	12-month price momentum	-3.5%	-2.9	-3.2%	-2.0
Momentum	3-month average mean EPS	-3.3%	-3.7	-2.9%	-2.5
Low risk	Historical beta	-3.2%	-2.4	-2.9%	-1.7

Sources: La Française Investment Solutions and JP Morgan

Whether they are labeled value, quality, momentum, or low risk, equity premia posted significant losses over these three days.

The criteria used by equity hedge fund managers to select stocks seem to be the same as those used to build equity alternative premia. This finding is no surprise: how does one select stocks if not by comparing their multiples (such as price/earning ratio), their profitability (such as return on equity), their price momentum (past 12-month return), and so on? In other words, the majority of long/short equity funds are exposed to alternative premia, and were exposed well before the label was invented.

The data in figure 7 requires an additional comment. Rebased to their respective standard deviations, the losses of the equity alternative premia during the quant crisis (between two and six times) are much smaller than those recorded by the hedge funds (13 times on average). But the reader must keep in mind the results discussed in the second section of this article. If the funds posted such huge losses, it is not only because the strategies they were exposed to registered negative returns but also – and especially – because strategies the manager expected to be de-correlated ended up displaying highly correlated returns.

Circumstances in which alternative premia are prone to re-correlation can occur at the worst possible time – when risk aversion is rising and risky assets move to the downside. The mixed results registered by alternative premia solutions since 2013 should prompt investors to question simulations that ignore such risks. ■

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[1] *Alternatively "risk premia", "style premia", "style factors", "risk factors", "factor premiums", etc.*
[2] *For the rest of the paper, we use the term of "premia", "alternative premia" or "strategy" whatever the underlying rationale.*
[3] *About the number of alternative premia that are often presented in the academic papers.*
[4] *Simulations performed by AQR and Deutsche Bank as detailed in the bibliography.*

Ten commandments for alternative premia investing

Luc Dumontier **sets out ten commandments for investors looking to construct a robust premia portfolio with stable performance**

1 Go beyond the academic

Most factor investing strategies – whether long-only ('smart beta') or long/short ('alternative premia'[1]) – are based on academic factors and seek to capture standard investment styles, including value, carry, momentum, low risk and so on, within traditional asset classes. The rush into factor investing strategies raises legitimate concerns that these common premia may become overvalued, and thereby structurally compressed and overcrowded, magnifying dislocation episodes such as 2007's quant crisis[2].

The best way to mitigate this risk is to broaden the scope of alternative premia.

The academic approach can be extended to other asset classes such as commodities (Dumontier and Garchery, 2015), corporate bonds (Houweling and van Zundert, 2014) and implied assets. The best-known example of the latter is the 'volatility premium', which seeks to monetise the spread between implied and realised volatility of a given asset. Strategies with different investment horizons to those of 'low-frequency' academic premia bring further diversification; for example, a 'pair trading' bet on the convergence between two historically correlated securities, typically over a period not exceeding a week.

Insurance-linked securities also offer interesting potential for alternative premia strategies. Indeed, insurance and reinsurance companies take on the role of the policyholder by assigning (life and non-life) risks to investors and paying them premiums. Finally, certain arbitrage strategies exploit pricing inefficiencies in the cash (or spot) and futures markets for the same asset, often due to the inability of market participants to hold the underlying asset.

Image: Shutterstock

2 Do not invent factors

The factor investing buzz has spurred a hunt for new strategies in a quest for diversification. And he who seeks shall find. Harvey, Liu and Zhu (2015) observed a strong increase in factor 'discoveries' since the seminal work of Sharpe on market beta in the 1960s (see figure 1). While the rate of factor discoveries was one per year on average in the 1980s, it increased to five in the 1990s and to almost 20 in the 2000s. To use the expression coined by Cochrane, the factor "zoo" now has several hundred factors.

Expertise in economics and/or statistics is not required to infer that most of these factors represent, at best, another expression of an existing factor (and is therefore likely to deliver correlated returns). At worst, they are unintelligible and probably unrepeatable; that is, unlikely to deliver returns over time. The onward rush of 'discoveries' is especially dangerous as the calibration error of a portfolio's volatility increases with the number of factors it includes, and soars if these factors – which are expected to be uncorrelated – re-correlate strongly[2]. To avoid inventing factors, each must fulfil the strict qualification criteria below.

3 Understand the underlying rationale

As per Warren Buffet, we should only invest in what we understand. What is true for stocks is even more so for alternative premia. Understanding the rationale underpinning each factor helps to ensure that: (i) they will persist so that each factor will continue to pay a premium, and (ii) they are different from one another so that factors will deliver uncorrelated premia[2].

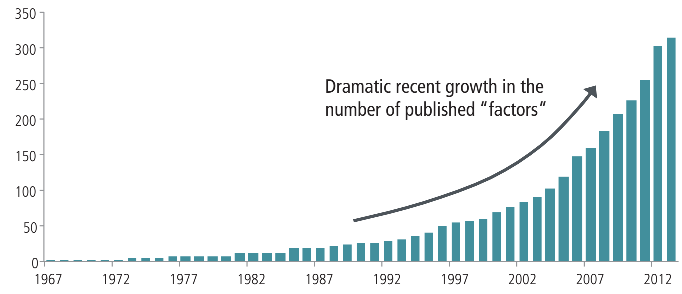
Alternative premia should only be retained if they either remunerate exposure to an additional risk factor that cannot be diversified away ('risk premia') or stem from biases linked to market participants' behaviour, investment constraints and structural flows ('style premia').

Thus selected, premia strategies are likely to persist. Rational investors will always require a return to take on additional risk. In the equity value strategy, for example, investors hold stocks with attractive valuations but which are, correspondingly, vulnerable to the 'value trap' phenomenon. Investors are paid a premium to assume this risk which could materialise if reasons for these low valuation multiples intensify. Similarly, behavioural biases are so strongly ingrained that it will always prove difficult for rational investors to arbitrage them completely. For example, investors tend to overreact in the short term to new information (eg, earnings publication). Mean-reverting strategies capitalise on this by buying past losers and selling past winners (using a lookback period of few days) to bet on the convergence in their short-term returns. Finally, regulation such as the Basel Accord for banks and Solvency Directive for insurance companies should generate more opportunities for non-constrained investors; for example, cash-and-carry arbitrage strategies.

4 Avoid data mining or over-fitting

While it is said that 'promises only bind those who believe in them', investors are often willing to trust simulations of factor-based strategies, assuming they are built using simple criteria supported by academic research. Nevertheless, Suhonen, Lennkh and Perez (2016) show

1. The factor zoo



Source: Harvey, Liu and Zhu (see references)

alternative beta strategies are far from immune to simulation biases. This comprehensive study analysed a wide range of rules-based strategies offered by investment banks, and found a median 73% deterioration in Sharpe ratios between back-tested and live performance periods (see figure 2). Interestingly, the fall-off in risk-adjusted performance was even greater for complex strategies with numerous rules and filters.

Recent research papers identify other common biases and help to separate the robust factors from the lucky factors. Harvey and Liu (2014) propose methods to account for multiple testing. Bailey and de Prado (2012) define the minimum track record needed for statistical significance. Amenc et al (2015) discuss the relative robustness or ability of a strategy to offer similar performance in similar market conditions. Investors should stick to strategies that resist parameter changes well, including the number of assets selected or the frequency of rebalancing (see figure 3).

5 Control exposure to underlying asset classes

It seems universally acknowledged that long/short portfolios that capture standard equity premia must be market (beta) neutral to preserve their diversification power, but little emphasis is placed on the importance of market neutrality for other asset classes.

For example, a carry premia strategy on foreign exchange is often implemented through a portfolio that is long the three highest-yielding currencies and short the three lowest yielding. The result is returns that are highly correlated with risky assets[2]. Similarly, a government bond portfolio that is long US and short Japanese bonds with the same duration displays positive overall market exposure, as US beta is far higher than that of Japan. Finally, a gold versus oil position is probably not 'commodity neutral'[3].

Investors should use principal component analysis (PCA) to control the biases to the underlying asset classes[3]. For example, developed market currencies (versus the US dollar) have common exposure to two factors that are robust over time (see figure 4). The 'US dollar factor' (x-axis) represents the co-movement of all currencies versus the US dollar. The 'bloc factor' (y-axis) represents the fact that dollar bloc commodity currencies on the one hand and European currencies on the other tend to display even stronger co-movements. According to this analysis, alternative premia should comprise positions such as 'AUD vs NZD' or 'SEK vs NOK' to be 'market neutral'. While the expected Sharpe ratios of these pairs is lower than the traditional forex '3 vs 3' carry trade, this is compensated for by low and stable correlation[2].

6 Control exposure to other alternative premia in the portfolio

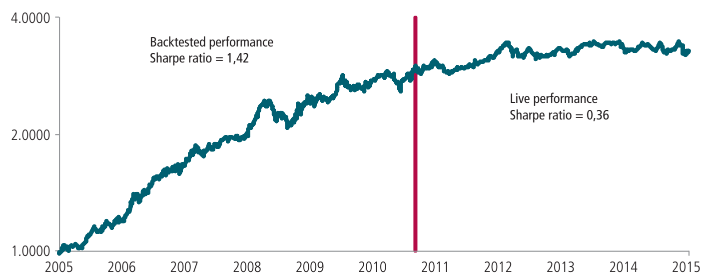
Even if biases versus main asset classes are controlled upstream (fifth commandment), premia may still be correlated – positively or negatively, structurally or cyclically. One topical bias is how expensive the low-risk equity premium is now, in terms of valuation multiples (eg, price-earnings and price-to-book ratios). This is often attributed to the popularity of this strategy and translates into cyclical negative exposure to the 'value vs growth' premium. The low-risk premium is also structurally negatively correlated to the 'small minus big' premium. Specifically, stocks of big companies – on average well diversified, both geographically and in terms of business mix – tend to be less volatile than the stocks of small companies.

The allocation process between premia (ninth commandment) can address this re-correlation risk. However, for the sake of parsimony and readability, we encourage a 'double-sorting' approach to build the purest possible premia strategy. As an illustration (see figure 5), the main biases of the low-risk premium can be minimised by: (i) removing from the investment universe the most expensive and cheapest stocks, and (ii) building several low-risk portfolios within each of the major capitalisation tranches (eg, big, medium and small).

7 Minimise idiosyncratic risks

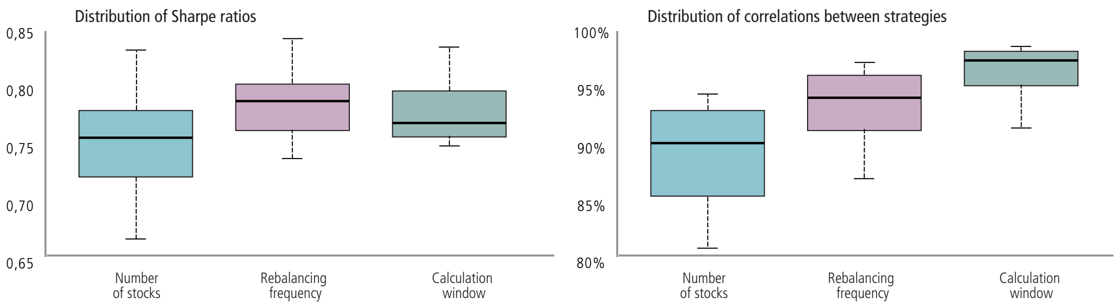
A major event in premia investing was the strong appreciation of the Swiss franc after the Swiss National Bank's decision to de-peg it from the euro in early 2015, and the subsequent simultaneous plunge of common forex academic premia[2]. Few realise events of this magnitude

2. Illustration of strategy performance
Backtest versus live sample (log scale)



Source: Suhonen, Lennkh and Perez (see references)

3. Ensuring the robustness of the equity low risk premia strategy



Source: La Française Investment Solutions

occur on a daily basis in equity markets; for example, following the news of a takeover bid or a profit warning. The equity universe being much larger than that of factors, equity portfolios are usually well diversified and less sensitive to strong idiosyncratic movements. The Swiss franc example serves as a useful reminder that a portfolio of alternative premia is, above all, a collection of individual positions and must be managed accordingly.

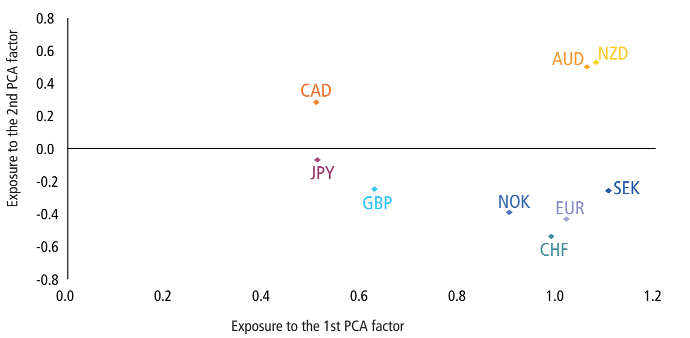
One approach is to underweight alternative premia based on asset classes where the investment universe is smaller. A better option would be to set ad hoc constraints in nominal terms to force the containment of idiosyncratic risks and expand the investment universe to the highest possible number of assets. As an example, many investment solutions implement premia in the government bonds space using only the four to five liquid 10-year futures. By using swaps, it is possible to more than double the number of underlying countries to which the strategy has exposure.

8 Monitor correlations in specific situations

Controlling historical correlation between premia (fifth and sixth commandments) and aggregate exposure to single assets (seventh commandment) does not mitigate concentration risk in full. For example, a portfolio with juxtaposed standard academic premia would have progressively carried significant 'commodity risk' in 2015: short commodity-related stocks (low risk and momentum premia), short high-yielding commodity currencies (momentum premium) and short energy commodities[3] (carry and momentum premia). If this risk is not addressed, the performance of the overall portfolio depends only on developments around this specific thematic – a significant departure from the diversification promised by 'factor investing'.

To gauge instantaneous correlation between premia, we suggest retropolating returns with current positions; that is, without any historical rebalancing. Simultaneous movements of these series, as well as the performance of the overall portfolio, particularly in response to: (i) periods of financial crisis (eg, Lehman bankruptcy), (ii) specific macroeconomic developments, (iii) strong movements in asset classes, and even (iv) customised scenarios are very useful for assessing concentration risk. The final step is to implement stop-loss policies. For example, if the current portfolio were likely to lose more than 5% in any considered scenario, a portion of the actual positions could be cut.

4. Principal component analysis (PCA) among G10 currencies

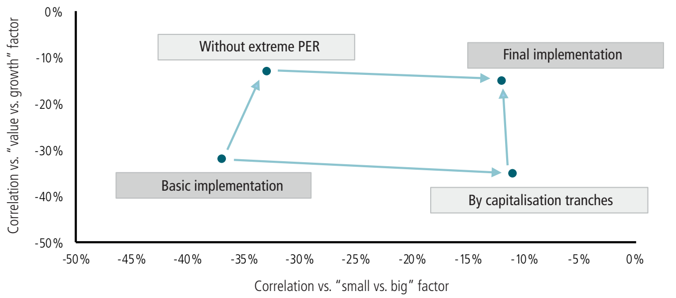


Source: La Française Investment Solutions

9 Beware of the temptation to time factors

According to Rob Arnott, founder and chairman of Research Affiliates, a Pimco subadvisor, many versions of smart beta equity products (eg, low volatility) became victims of their own popularity and grew increasingly expensive in terms of valuation multiples. This raised the question of whether factor timing can add value. In the other camp, Cliff Asness, co-founder of AQR Capital Management, found that timing strategies using the simple 'value' of the factors themselves did not deliver convincing results. The author's research

5. Orthogonalisation of the equity low risk premia strategy



Source: La Française Investment Solutions

supports the AQR view. This is unsurprising if we take a step back. If it is complicated to predict how equity markets will evolve, why should it be easier for alternative factors?

Furthermore, it is important to keep in mind that if a specific factor is excluded while maintaining the same target return for the portfolio, the remaining factors have to deliver individually higher Sharpe ratios to compensate for the resulting diversification shortfall. Removing one factor from an equally risk-weighted portfolio of five independent factors[4] would require the four remaining factors to each deliver a 20% higher Sharpe ratio to generate the same overall return – that seems unlikely. A more credible way of enhancing returns is to add new factors (first commandment), provided they comply with the selection criteria outlined above.

10 Invest in people and infrastructure

Compliance with the first nine commandments requires an investment team able to deploy experience and techniques from across the finance industry, including quantitative asset management and investment banking. A robust investment infrastructure is also necessary.

The investment team must be capable of identifying opportunities, as well as designing, implementing and managing a wide range of alternative premia, from academic to investment-banking strategies (first commandment). While different in nature, each strategy must respect the same set of selection criteria (second, third and fourth commandments) to maintain the coherence of the whole. They must also be built and combined to maximise diversification (fifth and sixth commandments), whatever the market context (eight commandment), while minimising specific risks (seventh commandment).

Efficient implementation is also important. Academic premia are mostly implemented using plain vanilla instruments. Here, every basis point counts and the ability to pre-negotiate the lowest possible transaction costs can have a significant impact. For premia implemented using derivatives instruments, dealing arrangements with the maximum number of counterparties is a determinant of success. Indeed, most of these investment strategies are only visible to investors whose scope of counterparty relationships allows them to see opportunities, such as a bank needing to recycle a given risk.

When solicited on the subject of smart beta and, by extension, alternative premia strategies, Markowitz is said to have compared this investment framework to so-called all-natural food at a grocery store. Many products may bear the 'smart beta' label; however, not all are necessarily all natural or even good for you. Each alternative premia strategy must be evaluated individually on its merits. ■

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[1]. *Alternatively, 'risk premia', 'style premia', 'style factors', 'risk factors', 'factor premiums', etc.*
[2]. *See 'Why re-correlation matters in alternative premia investing' by Dumontier, published on Risk.net, October 2016*
www.risk.net/2473808.
[3]. *See 'Commodity premia – It's all about risk control' by Dumontier and Garchery, published on Risk.net, December 2015*
www.risk.net/ 2437558
[4]. *About the number of independent factors that can be captured from the set of traditional academic premia.*

A triptych approach for reverse stress testing of complex portfolios

Pascal Traccucci, Luc Dumontier, Guillaume Garchery and Benjamin Jacot present an extended reverse stress test (ERST) triptych approach with three variables: level of plausibility, level of loss and scenario. Any two of these variables can be derived, provided the third is given as input. A new version of the Levenberg-Marquardt optimisation algorithm is introduced to derive the ERST in certain complex cases

Introduction: the case of ARP portfolios

Academic theory has been mined to support the development of investment solutions containing an ever-increasing number of factors. Over the last decade, academics and practitioners have shown traditional asset classes offer limited diversification, especially in market downturns. In response, they have delved into modern portfolio theory (MPT) to identify the microeconomic factors that are the backbone of alternative risk premia (ARP) solutions. The ARP 1.0 approach combines 10–15 different long/short portfolios capturing standard investment styles such as value, carry, momentum, low risk and liquidity across a broad range of traditional asset classes. For further diversification, the ARP 2.0 approach combines up to 30 strategies by including investment banking-style premia likely to use instruments with quadratic profiles.

Many risk management frameworks cannot properly account for nonlinear profiles and assess the risk of loss associated with combining an unusually high number of strategies. Specifically, historical value-at-risk is an instantaneous risk indicator and does not correspond to a clearly identified scenario; hence the need for complimentary stress tests. To build a stress-testing tool, the dataset must be simplified, and historical or predefined scenarios are used without quantifying their plausibility. Thus, parametric VAR imposes dependence on a model to benefit from an analysis framework in the form of a VAR and a sensitivity of this VAR to all the parameters of the model. This requires several numerical problems to be addressed, especially in case of quadratic profit and loss (P&L). This article presents an innovative approach: the extended reverse stress test (ERST), following on from the work of Breuer *et al* (2009) and Mouy *et al* (2017). This approach is able, with low technical costs,[1] to deliver two of three parameters, provided the third is given as input. The three parameters are scenario, level of plausibility and level of loss (see figure 1). The result is a more meaningful risk measure and one that corresponds to a clearly identified scenario.

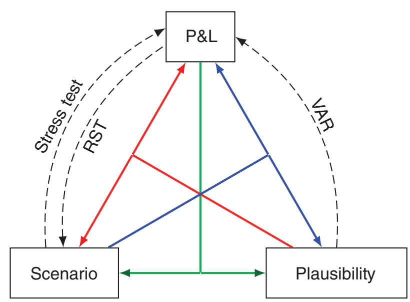
In what follows, $\boldsymbol{S}$ is defined as a scenario. It is a vector with length n, which equals the number of risk factors to which the portfolio is exposed. In addition, the covariance matrix of the risk factors will be denoted by $\boldsymbol{\Sigma}$.

Starting from a scenario

A scenario-driven ERST approach is suitable for a portfolio manager considering a given adverse or best-case scenario $\boldsymbol{S}_0$. To assess the plausibility of such a scenario, the probability α_0 of a scenario being as extreme as or less extreme than $\boldsymbol{S}_0$ is computed. If α_0 is too high, $\tilde{\boldsymbol{S}}$, a more plausible scenario than $\boldsymbol{S}_0$, is derived and suggested to the portfolio manager.

[1] *Using an algorithm derived from the Levenberg-Marquardt one to deal with complex problems.*

1 The triptych approach of the extended reverse stress test (ERST)



Measuring plausibility. The ERST relies heavily on the concept of plausibility (or likelihood) to discriminate between the scenarios generated. Multiple plausibility measures exist in the literature (Breuer *et al* 2009). In this article, plausibility is quantified in terms of the Mahalanobis distance. The latter measures the amplitude of the multivariate moves in $\boldsymbol{S}$ from the mean scenario $\boldsymbol{\mu}$ in units of standard deviation. It is therefore similar in a multidimensional space to the concept of a Z-score z or standardised variables. As a reminder:

$$z = \frac{x - \mu_X}{\sigma_X}$$

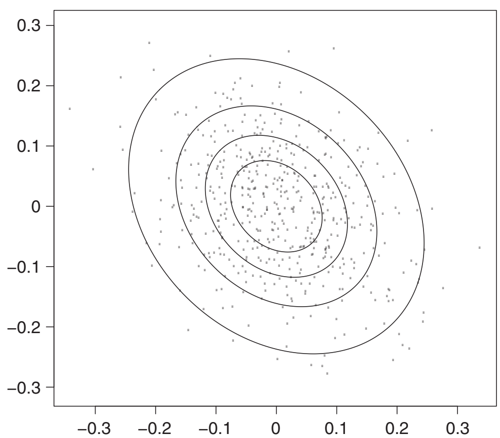
where x is the realisation of a random variable X with mean μ_X and standard deviation σ_X. The Mahalanobis distance is defined as follows:

$$\mathrm{Maha}^2(\boldsymbol{S}) = (\boldsymbol{S} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{S} - \boldsymbol{\mu}) \tag{1}$$

Unlike other measures, the Mahalanobis distance is both intuitive and simple to use. Its following characteristics are noteworthy:

- A low (respectively, high) Mahalanobis distance characterises a highly plausible (respectively, unlikely) scenario.
- $\mathrm{Maha}^2(\boldsymbol{S}) = R^2$ is the surface of an ellipsoid of radius R. Points within the ellipsoid have a Mahalanobis distance of less than R. The further away these points are from the surface, the closer they are to the centre, and the more plausible they become.
- Assuming $\boldsymbol{S}$ follows a multivariate normal distribution, $\mathrm{Maha}^2(\boldsymbol{S})$ follows a $\chi^2(n)$ distribution, as proved in Studer (1997). The α quantile of a $\chi^2(n)$ density is thus the squared radius of the ellipsoid, where $\alpha\%$ of the multivariate normal scenarios $\boldsymbol{S}$ remain inside. Hereafter, this ellipsoid is referred to as $\mathcal{E}^{\alpha}$. See figure 2 as an illustration. Also, α is referred to as

2 Plausibility domains for a bivariate random variable with elliptical density



The inner (respectively, outer) ellipse corresponds to a 25% (respectively, 95%) quantile. The in-between ellipses correspond to 50% and 75% quantiles

the probability a scenario is as plausible as or more plausible than $\boldsymbol{S}$, or the probability of non-occurrence.

■ Mahalanobis distance is suited to any elliptical multivariate distribution for $\boldsymbol{S}$, which includes densities other than multivariate normal, for example, Student's t distribution. This is of primary importance as a distribution of this type is typically a better fit for historical distributions than a normal one, especially as concerns fat tails.

The plausibility of $\boldsymbol{S}_0$ can now be evaluated simply by using (1). Assuming a normal distribution, the resulting value is compared with the quantiles of a $\chi^2(n)$ to determine the probability of a scenario as extreme as or less extreme than $\boldsymbol{S}_0$. For other elliptical distributions where the law of the Mahalanobis distance is not known, a numerical solution exists. First, the elliptical distribution that best fits $\boldsymbol{S}$ is determined. Second, a Monte Carlo simulation of $\boldsymbol{S}$ is performed. Then, approximate quantiles of the Mahalanobis distance are computed and the probability of non-occurrence $\alpha_{0,\text{approx}}$ of $\boldsymbol{S}_0$ can be deduced.

■ **Fitting the plausibility of a given scenario.** If α_0 or $\alpha_{0,\text{approx}}$ exceeds a given threshold $\alpha_{\max}$, then $\boldsymbol{S}_0$ lies outside of the admissible ellipsoid $\mathcal{E}^{\alpha_{\max}}$. In this case, the closest admissible scenario $\tilde{\boldsymbol{S}}$ to $\boldsymbol{S}_0$ on $\mathcal{E}^{\alpha_{\max}}$ is defined by uniform scaling. This definition results in minimal corrections with regard to $\boldsymbol{S}_0$ and thus adheres as closely as possible to the intuition of the portfolio manager.

For the sake of clarity, the non-constraining assumption $\boldsymbol{\mu} = \mathbf{0}$ is made. As $\tilde{\boldsymbol{S}} \in \mathcal{E}^{\alpha_{\max}}$, it follows that $\tilde{\boldsymbol{S}}^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \tilde{\boldsymbol{S}} = q^{\alpha_{\max}}$, where $q^{\alpha_{\max}}$ is the $\alpha_{\max}$ quantile of the density of the squared Mahalanobis distance. This constraint leads to:

$$\tilde{\boldsymbol{S}} = \sqrt{q^{\alpha_{\max}}} \frac{\boldsymbol{S}_0}{\sqrt{\boldsymbol{S}_0^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \boldsymbol{S}_0}} \tag{2}$$

■ **Application.** A portfolio manager runs two long/short strategies, each based on a different spread: the first on equity indexes (S&P 500 versus Euro Stoxx 50) and the second on bonds (German Bund versus US Treasury). Analysis of monthly prices from the previous five years (2014–19) shows low correlation ρ between spreads: $\rho \approx 0.01$. Over the same period, the spread returns have a monthly volatility of 3.3% and 1.2%, respectively.

The manager would like to know if, under these assumptions, there is a strong probability (50%, for example) the spread scenarios will incur 1.5% and 2.5% losses over one month, leading to the scenario:

$$\boldsymbol{S}_0 = [-1.5\%, -2.5\%] \tag{3}$$

In this example, it is assumed the spreads have either a normal or a Student's t distribution. The parameters of the elliptical distribution of reference are determined using maximum likelihood estimators derived from the historical distribution. Thus, $\boldsymbol{S}_0$ corresponds to $\alpha_0 = 91\%$ (respectively, 81%) for a normal (respectively, a Student's t) distribution. By way of comparison, the average monthly market correction observed during the fourth quarter of 2018 shows a probability of non-occurrence of approximately 77% (respectively, 69%). Therefore, the loss the manager had in mind is less plausible than expected. Setting $\alpha_{\max} = 50\%$ in (2), the fitted scenario of interest for the manager is:

$$\begin{aligned} \tilde{\boldsymbol{S}} &= [-0.8\%, -1.3\%] \quad \text{for normal risk factors} \\ &= [-0.9\%, -1.4\%] \quad \text{for Student's } t \text{ risk factors} \end{aligned} \tag{4}$$

Thus, the fitted scenarios respect the directions intended by the portfolio manager, and only the amplitude of the shocks is changed to comply with the constraint $\alpha_{\max}$.

Obviously one can argue the correlation and volatility used in this example do not reflect a crisis environment where $\boldsymbol{S}_0$ occurs. An extension of this example would therefore be to stress the correlation and volatility to best reflect a financial crisis environment. This process is further explained in Traccucci *et al* (2019).

Starting from plausibility

The plausibility-driven ERST returns both the most extreme loss and a corresponding scenario for a given level of plausibility. This approach is studied in Studer (1997) and further discussed in Breuer *et al* (2009), for example. Its advantage is it returns a loss that may be compared with other existing risk measures such as VAR, which is briefly introduced in the following subsection. As shown in the rest of this section, a plausibility-driven ERST is linearly dependent on VAR for linear and some non-linear portfolios. However, this relationship is not present as a general rule for non-linear portfolios, making plausibility-driven ERST interesting and valuable. For non-linear portfolios, the approach can be seen as a continuum of VAR and expected shortfall (ES), and it sets a new paradigm for risk measurement. Some limitations do exist, however, as discussed at the end of this section.

■ **Existing VAR approach.** For a given $\alpha \in [0, 1]$, VAR_α returns the α quantile of the P&L density, indicating the P&L is not as extreme as the VAR output $\alpha\%$ of the time. The P&L density may be a historical or any other fitted density.

Taking the simple case of a linear portfolio with n risk factors and a weighting scheme $\boldsymbol{\omega}$, and assuming the risk factors are normally distributed $\boldsymbol{S} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$, then $\text{P\&L}(\boldsymbol{S}) \sim \mathcal{N}(0, \boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{\omega})$ and:

$$\mathrm{VAR}_\alpha = -\mathcal{N}^{-1}(\alpha) \sqrt{\boldsymbol{\omega}^{\mathrm{T}} \boldsymbol{\Sigma} \boldsymbol{\omega}} \tag{5}$$

for $\mathcal{N}^{-1}(\alpha)$, the α quantile of a standard normal distribution.

For a quadratic portfolio, this expression does not hold true. The distribution of P&L($\boldsymbol{S}$) may not be analytically known depending on the density function for $\boldsymbol{S}$. However, an approximate VAR can be calculated after Monte Carlo simulations of $\boldsymbol{S}$ and a derivation of the probability based on the resulting P&L distribution.

With such an approach, VAR provides only one output: a loss. This does not allow a portfolio manager to dig deeper and understand where the underlying weaknesses in portfolio exposures lie.[2] In this respect, the plausibility-driven ERST provides a more complete result than VAR. In addition to a resulting loss, it provides a corresponding scenario, identifying the specific strengths and weaknesses of the portfolio. This allows the portfolio manager to take potential countermeasures such as hedging or portfolio adjustments. This advantage is discussed in more detail below.

Problem statement. Let α and MaxERST[3] be the input level of plausibility and the output loss, respectively. The plausibility-driven ERST is then the optimisation problem:

$$\min_{\mathrm{Maha}^2(\boldsymbol{S})\leqslant q^\alpha} \mathrm{P\&L}(\boldsymbol{S}) \tag{6}$$

In the two following sections, this problem is solved for both linear and quadratic portfolios.

Application for delta-one strategies. Here, (6) can be solved by relying on Lagrangian optimisation with Kuhn-Tucker conditions. For a given α, MaxERST and the corresponding scenario $\boldsymbol{S}^\alpha$ are:

$$\boldsymbol{S}^\alpha = -\sqrt{q^\alpha}\frac{\boldsymbol{\Sigma\omega}}{\sqrt{\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Sigma\omega}}} \tag{7}$$

$$\mathrm{MaxERST} = -\sqrt{q^\alpha}\sqrt{\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Sigma\omega}} \tag{8}$$

Figure 3 shows an application to a long/short strategy on two momentum indexes, with $\boldsymbol{\omega} = (1, -1)$. Comparing (5) and (8), MaxERST and VAR are proportional. For linear portfolios, Breuer (2006) states a similar relationship, adding that VAR and MaxERST are also proportional to the ES measure. The corresponding proof is by Sadefo-Kamdem (2004). Therefore, when $\boldsymbol{S}$ is normally distributed:

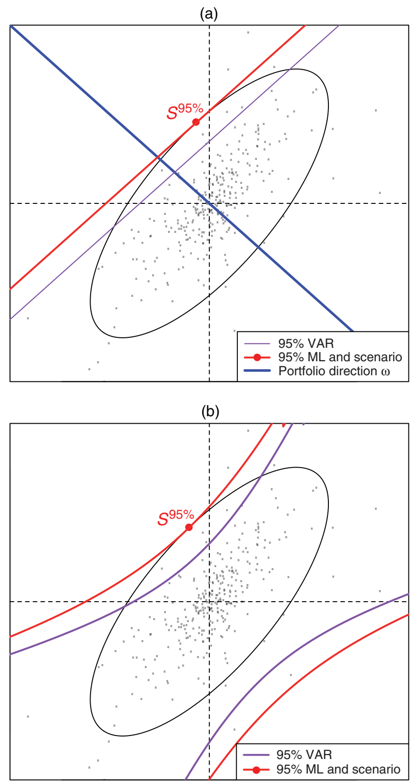
$$\frac{\mathrm{VAR}}{\mathcal{N}^{-1}(\alpha)} = \frac{\mathrm{MaxERST}}{\sqrt{q^\alpha}} = \frac{\mathrm{ES}}{\rho(\alpha)\alpha} = -\sqrt{\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\Sigma\omega}} \tag{9}$$

where q^α is the α quantile of a $\chi^2(n)$ distribution and $\rho(\alpha)$ is the density of the standard normal distribution (Breuer 2006). Despite being proportional for delta-one strategies, Breuer (2006) argues MaxERST is more useful than VAR. As it is sub-additive[4] whereas VAR is not, MaxERST has proved to be a more reliable limit system than VAR for simple non-linear portfolios such as some combinations of out-of-the-money short puts and short calls on the same underlying.

For q^α, the quantile of a $\chi^2(n)$ distribution $\lim_{n\to\infty} q^\alpha = \infty$. Therefore, if $\boldsymbol{S}$ is normally distributed, the higher the number n of risk factors to which the portfolio is exposed, the more extreme MaxERST will be relative to VAR as per (9). This could create a dimensional dependency issue for irrelevant factors, as exposed in Mouy *et al* (2017) or Breuer *et al* (2009) and further discussed in the subsection titled 'On dimensional dependency' hereafter.

[2] *This is possible with historical VAR but only for historical/past scenarios.*
[3] *MaxERST was first introduced by Studer (1997) and denoted as maximum loss (ML).*
[4] *By which we mean absolute losses are such that MaxERST(portfolio* 1) + *MaxERST(portfolio* 2) ⩾ *MaxERST(portfolios* 1 + 2) ⩾ 0.

3 Plausibility-driven ERST for (a) a linear or (b) a non-linear portfolio



300 data points for two momentum indexes (in grey) are used to compute $\boldsymbol{\Sigma}$. The level of plausibility is fixed at $\alpha = 95\%$ and $\mathcal{E}^{95\%}$ is in black. Knowing the VAR and MaxERST as per (5) and (8), their corresponding iso-P&L lines are indicated

Application for non-linear P&L. For non-linear P&L, a second-order approximation is considered. Thus:

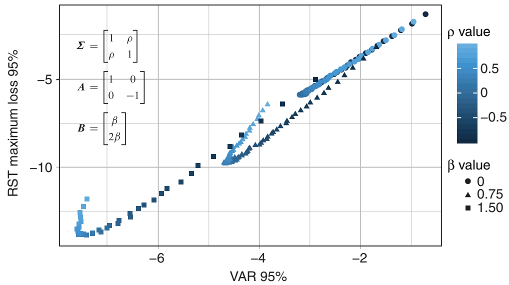
$$\mathrm{P\&L}(\boldsymbol{S}) = \tfrac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{AS} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{S} \tag{10}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are the second- and first-order sensitivities of the portfolio, respectively. Second-order sensitivities being symmetric, $\boldsymbol{A}$ is symmetric. With a quadratic form for P&L, the resolution of (6) is more complex. The objective function may not be convex, therefore Kuhn-Tucker conditions are irrelevant. Fortunately, there is an optimisation algorithm that can cope with this issue: the Levenberg-Marquardt algorithm. This is introduced in depth by Nocedal & Wright (1999) and applied by Studer (1997) to solve (6) with (10).

For the same two momentum indexes as in the previous subsection, results are shown in figure 3 for some $\boldsymbol{A}$ and $\boldsymbol{B}$.

In addition, MaxERST is no longer linear with respect to VAR, as opposed to delta-one strategies. This result justifies the use of ERST rather than VAR for non-linear portfolios, as the approach offers added value to the portfolio manager and is a continuum of VAR. Figure 4 shows the aforementioned

4 Comparison between VAR and MaxERST (or maximum loss) outputs for $\alpha = 95\%$ and two risk factors



Two risk factors are simulated using unitary volatility for simplicity. For $\beta \neq 0$, the linear relation between VAR and MaxERST disappears. For $\beta = 1.5$, VAR does not vary much with the positive correlation, whereas MaxERST does. The latter is therefore a more interesting risk measure, as it reacts more strongly to changes in correlation

non-linearities. And yet the specific case where $\boldsymbol{B} = \boldsymbol{0}$ remains linear, as proved in Traccucci *et al* (2019, appendix 1).

On dimensional dependency. As stressed in Mouy *et al* (2017) and Breuer *et al* (2009), the output of this ERST approach depends directly on the dimension of the problem, ie, the number n of risk factors under consideration. Indeed, q^α in (6) varies with n. As previously stated, q^α is, for example, the quantile of a $\chi^2(n)$ distribution for a normally distributed $\boldsymbol{S}$.

Although this may be viewed as a source of instability, it is also positive from a portfolio management perspective: it is actually a way to account for correlation with the external yet meaningful risk factors that indirectly drive variations.

In addition, to bypass the instability caused by dimensional dependency, Rouvinez (1997) suggests replacing q^α with the Mahalanobis distance of a given scenario.

Starting from P&L

A P&L-driven ERST extends the ideas expressed by Mouy *et al* (2017) to non-linear portfolios. To this end, a new, adapted version of the Levenberg-Marquardt optimisation algorithm is defined and tested. The main advantage of such an approach as compared with starting from plausibility is to overcome the aforementioned dimensional dependency issue.

Problem statement. Given the dimensional dependency issue, it is preferable the constraint in (6) be independent of the squared Mahalanobis quantiles. Here, inverting the problem formulation works, ie, finding the scenario with optimal plausibility for a given P&L. This paves the way for the third and final approach discussed in this article. The optimisation problem becomes:

$$\min_{\mathrm{P\&L}(\boldsymbol{S})=l} \mathrm{Maha}^2(\boldsymbol{S}) \tag{11}$$

The case for a linear P&L is discussed in Mouy *et al* (2017), but the resolution for non-linear P&L remains outstanding. The remainder of this section focuses on this.

Resolution for non-linear P&L. Rewriting (11) brings, for a loss l:

$$\min_{\frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{S}+\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\leqslant l} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{S} = \min_{\frac{1}{2}\hat{\boldsymbol{S}}^{\mathrm{T}}\hat{\boldsymbol{A}}\hat{\boldsymbol{S}}+\hat{\boldsymbol{B}}^{\mathrm{T}}\hat{\boldsymbol{S}}\leqslant l} \|\hat{\boldsymbol{S}}\|^2 \tag{12}$$

where the change of variable $\hat{\boldsymbol{S}} = \boldsymbol{U}^{-\mathrm{T}}\boldsymbol{S}$ is performed with $\boldsymbol{U}$, the Cholesky decomposition matrix for $\boldsymbol{\Sigma}$, and:

$$\hat{\boldsymbol{A}} = \boldsymbol{U}\boldsymbol{A}\boldsymbol{U}^{\mathrm{T}} \tag{13a}$$

$$\hat{\boldsymbol{B}} = \boldsymbol{U}\boldsymbol{B} \tag{13b}$$

Changing the variable allows the quadratic optimisation problem to work with a centred bowl rather than an ellipsoid. The problem is thus reduced to finding the closest scenario(s) $\hat{\boldsymbol{S}}^*$ to the origin and associated with the iso-loss curve of value l. This problem relates to the Levenberg-Marquardt optimisation problem used when starting from a plausibility. However, the constraint is not necessarily convex here. Therefore, a new version of the method is introduced.

It can be derived[5] from the equivalence proved in Nocedal & Wright (1999, theorem 4.3) that $\hat{\boldsymbol{S}}^*$ is a solution to (12) if, and only if, it verifies the following conditions for λ_m, the smallest eigenvalue of $\hat{\boldsymbol{A}}$, and a given μ:

$$(\hat{\boldsymbol{A}} + \mu\boldsymbol{I})\hat{\boldsymbol{S}}^* = -\hat{\boldsymbol{B}} \tag{14a}$$

$$\mu(\tfrac{1}{2}\hat{\boldsymbol{S}}^{*\mathrm{T}}\hat{\boldsymbol{A}}\hat{\boldsymbol{S}}^* + \hat{\boldsymbol{B}}^{\mathrm{T}}\hat{\boldsymbol{S}}^* - l) = 0 \tag{14b}$$

$$\mu \geqslant \max(0, -\lambda_m) \tag{14c}$$

The multidimensional optimisation problem (12) reduces to a scalar optimisation problem on μ under constraints (14a)–(14c). A problem of this type can be solved rapidly. As detailed below, a bisection algorithm to find the optimal μ allows for $\hat{\boldsymbol{S}}^*$ to be inferred directly.

If $\mathcal{B} = (\boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_n)$ is an orthonormal diagonalising basis of symmetric matrix $\hat{\boldsymbol{A}}$, then:

$$\hat{\boldsymbol{S}}^* = \sum_i \sigma_i \boldsymbol{\pi}_i \tag{15}$$

$$\hat{\boldsymbol{B}} = \sum_i \beta_i \boldsymbol{\pi}_i \tag{16}$$

Defining $(\lambda_i)_i$, the eigenvalues of $\hat{\boldsymbol{A}}$, $I_m = \{i, \lambda_i = \lambda_m\}$, and taking (14c) into account, (14a) expressed in $\mathcal{B}$ becomes:

$$\sigma_i = -\frac{\beta_i}{\lambda_i + \mu} \qquad \forall i \notin I_m \tag{17a}$$

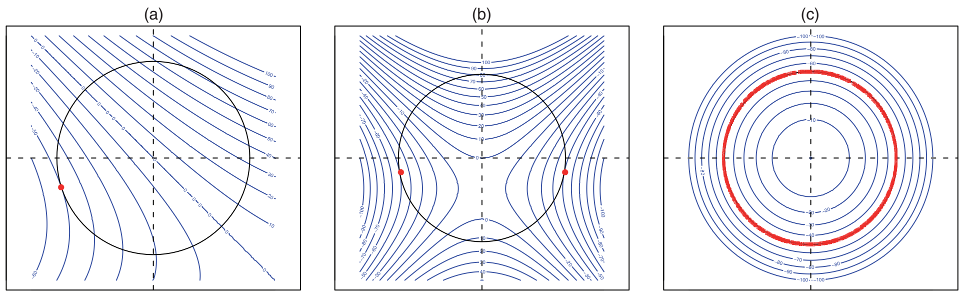
$$\sigma_j = -\frac{\beta_j}{\lambda_m + \mu} \qquad \forall j \in I_m \text{ if } \mu \neq -\lambda_m \tag{17b}$$

$$\sigma_j \in \mathbb{R} \quad \text{and} \quad \beta_j = 0 \qquad \forall j \in I_m \text{ if } \mu = -\lambda_m \tag{17c}$$

It is thus possible to find a unique $\hat{\boldsymbol{S}}^*$ if (17b) is met, and several $\hat{\boldsymbol{S}}^*$ parameterised by $(\sigma_j)_{j\in I_m}$ if (17c) is met. This result is important, because it illustrates the different cases of existence and unicity of $\hat{\boldsymbol{S}}^*$. In this respect, it is more complex than the functional expression obtained for the original

[5] *This article does not provide rigorous proof of this statement, resembling that in Nocedal & Wright (1999) for convex P&L. Instead, for both clarity and applicability, this article shows the statement solves all of the variations the optimisation problem (11) takes.*

5 Solutions to (12) when $\hat{\boldsymbol{S}}$ consists of two risk factors



Depending on the P&L expression, (a) one solution, (b) two solutions or (c) an infinity of solutions are found

Levenberg-Marquardt problem analysed by Nocedal & Wright (1999) and Studer (1997).

Expressing (14b) in $\mathscr{B}$ brings $\mu(f(\mu) - l) = 0$ with:

$$f(\mu) = \sum_i \left[\frac{\lambda_i}{2} \left(\frac{\beta_i}{\lambda_i + \mu} \right)^2 - \frac{\beta_i^2}{\lambda_i + \mu} \right] \qquad \text{if } \mu \neq -\lambda_m \quad (18\text{a})$$

$$= \sum_{i \notin I_m} \left[\frac{\lambda_i}{2} \left(\frac{\beta_i}{\lambda_i - \lambda_m} \right)^2 - \frac{\beta_i^2}{\lambda_i - \lambda_m} \right] + \frac{\lambda_m}{2} \sum_{j \in I_m} \sigma_j^2 \qquad \text{if } \mu = -\lambda_m \quad (18\text{b})$$

These different dynamics lead to the following discussion on $\hat{\boldsymbol{A}}$:
(1) For $\hat{\boldsymbol{A}}$ positive definite, $-\lambda_m < 0$ and $\mu \geqslant 0$ as per (14c).
(1a) If $\mu = 0$, $\hat{\boldsymbol{S}}^* = -\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}$ as per (14a) and:

$$\begin{aligned} \text{P\&L}(\hat{\boldsymbol{S}}^*) &= -\tfrac{1}{2}\hat{\boldsymbol{B}}^{\mathrm{T}}\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}} \\ &= -\tfrac{1}{2}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{B}, \end{aligned}$$

which corresponds to the global minimum P&L. Such a value of μ is chosen whenever the scenario for the global minimum is more plausible than the most plausible scenario for loss l.
(1b) If $\mu \neq 0$, then the loss l is attained as per (14b). However, such a loss must be greater than the global minimum P&L. If f is continuous and increasing, a single μ corresponds to any loss and can be approximated using a bisection algorithm.
(2) For $\hat{\boldsymbol{A}}$ semi-positive definite, $-\lambda_m = 0$ and $\mu \geqslant 0$.
(2a) If $\mu = 0$, (17c) applies. As per (18b), the P&L does not vary with any σ_j, $j \in I_m$, and the corresponding risk factors become irrelevant. The dimensions of the problem are thereby reduced and it becomes similar to (1a).
(2b) If $\mu \neq 0$, then loss l is attained as per (14b). Since $\lim_{-\lambda_m^+} f = -\infty$ and $\lim_{+\infty} f = 0$, and f is still continuous and increasing, a single μ corresponds to any loss and can again be approximated using a bisection algorithm.
(3) For any other $\hat{\boldsymbol{A}}$, $-\lambda_m > 0$ and $\mu \geqslant -\lambda_m$.
(3a) If $\mu = -\lambda_m$, (17c) applies. As per (18b), the P&L still varies with σ_j, $j \in I_m$. Constraint (14b) becomes $f(\mu) = l$ and a root-finding algorithm (such as Newton-Raphson) can determine which values σ_j, $j \in I_m$, must take. This solution may or may not be unique.
(3b) If $\mu \neq 0$, (2b) applies.

This indicates that it is only possible to solve (12) for losses ($l \leqslant 0$). This is actually a direct consequence of the formulation of the problem itself. Indeed, the null scenario always returns a zero-valued P&L per (10). In addition, the null scenario returns the lower boundary of the objective function in (12). Therefore, a profit input ($p > 0$) cannot be obtained as a null scenario both returns a lower value in the objective function and respects the P&L constraint. However, generating profit scenarios is of significant interest in assessing the asymmetries in portfolio P&L. Thus, for $p > 0$, (12) may be rewritten as follows:

$$\min_{-[\frac{1}{2}\hat{\boldsymbol{S}}^{\mathrm{T}}\hat{\boldsymbol{A}}\hat{\boldsymbol{S}} + \hat{\boldsymbol{B}}^{\mathrm{T}}\hat{\boldsymbol{S}}] \leqslant -p} \|\hat{\boldsymbol{S}}\|^2 \qquad (19)$$

Application to non-linear P&L. The adapted Levenberg-Marquardt algorithm is tested on portfolios with two risk factors in figure 5. This algorithm is not sensitive to the number of risk factors, as the numerical procedure for solving (12) is reduced to analysing the function f of one variable in (18). Therefore, the numerical techniques involved are time-efficient.

Conclusion

The ERST adds value compared with traditional stress tests and risk measures such as VAR or ES, mostly because its output contains more information. This additional information can help both portfolio and risk management teams to control a portfolio's sensitivities and reallocate resources as and when needed.

Possible next steps may include:

- A procedure for recomputing Greeks in (10) to better account for their potential instability in scenarios with high market moves.
- A bootstrapping procedure for the covariance matrix $\boldsymbol{\Sigma}$ in (1). This would mitigate the error in the estimated plausibility of a scenario due to the estimation of $\boldsymbol{\Sigma}$.
- A procedure for better interpreting any ERST output scenario. An interesting starting point may be the maximum loss contribution defined by Breuer *et al* (2009).

■ A procedure to go beyond the restriction of multivariate elliptical distributions. The use of copulas as done by Mouy *et al* (2017) would serve as a starting point.

■ A procedure to stress $\boldsymbol{\Sigma}$. Because of (1), the covariance matrix affects the stressing of the portfolio, which is of primary importance in risk management. In this respect, two methods to stress $\boldsymbol{\Sigma}$ are provided and illustrated in Traccucci *et al* (2019). They account for the risk of recorrelation between supposedly independent strategies in market downturns. The influence of the covariance matrix in generating the most plausible scenarios is also discussed. ■

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